Gdańsk University of Technology
Faculty of Electronics,
Telecommunications and Informatics

Department: **Katedra Systemów Geoinformatycznych**
Author: **Tomasz Mrugalski**
Studies type: **M. Sc. Studies**

M. Sc. Dissertation

Dissertation topic:

Trajectory determination and orbital maneuvers in Earth vicinity

Polish title:

Wyznaczanie trajektorii i manewry orbitalne w otoczeniu Ziemi

Supervisor: **prof. dr hab. inż. Cezary Specht**
Technical Supervisor: **dr inż. Mariusz Specht**

Gdańsk, 2020-09-27

Contents

1. Introduction

This chapter provides introduction to the topic of astrodynamics, history of its development, introduces modern notations, reviews orbit types, and orbital maneuvers.

1.1. Astrodynamics

Orbital mechanics (*pol.* mechanika orbitalna), or astrodynamics (*pol.* astrodynamika), is a branch of applied science concerned with the study of motion of natural (planets, asteroids, comets, etc.) and man made bodies (probes, rockets, etc.) in outer space. The motion of such objects is most typically predicted using Newton's laws of motions and law of universal gravitation [25], but other forces, such as propulsive maneuvers, solar pressure, Earth oblateness, solar wind and other phenomena are taken into consideration as well.

1.2. Thesis overview and goals

The primary goal of this thesis is to provide a framework for solving selected problems in astrodynamics. The envisaged applications are two fold. First, the environment together with this thesis, can be used to conduct research in related topics. Second, the studied problems are chosen in a way so they could be directly applicable to space missions being considered in Poland in the near future.

Problem 1: Georeferencing satellite images. In a separate project, author constructed satellite ground station that is able to receive weather satellite images from NOAA satellites. The data acquired is raw and due to frequent cloud cover, the images are hard to read. It would be beneficial to perform georeferencing and overlay country and land contours. This requires calculating reference position of an image obtained from a Earth observation satellite, based on its known properties (orbital elements, sensor dimensions, optical properties), converting to geodetic reference systems, such as WGS-84, and then overlaying country borders, grid and other types of data.

Problem 2: Reviving or deorbiting old satellites. Many older satellites still have fully functional electronic and optical systems, but they are no longer able to conduct their primary missions due to running out of fuel. The idea here is to propose a trajectory for a small satellite that would perform rendezvous maneuver and attach to an old dysfunctional satellite and act as its strap-on engine. This can be used to either regain control over the satellite if its on-board systems are still functional or deorbit if they aren't. This problem focuses on on-orbit navigation. It requires designing several maneuvers: achieving orbit after launch, inclination change, orbit raise using Hohmann transfer, chase and rendezvous maneuvers.

Problem 3: Debunking fake news. News media are often publishing alarming articles about the upcoming asteroids close fly-bys with with varying levels of inaccuracy and fake sensationalism. The recent close fly-by of asteroid 1998 OR-2 on 29th April 2020 was a good example. The goal of this problem is to present an easy to follow step by step explanation how to calculate ephemerides for the upcoming close passes of asteroids and comets, including current known uncertainties and compare it to predicted closest distance.

Problem 4: Interplanetary transfer windows. The difficulty of reaching a place in a Solar system is not expressed in distance, but in the relative change of velocity needed to get there. Due to the bodies being in constant movement, the difficulty changes over time. There are certain configurations where reaching one planet from another are more easier. Such periods of favorable configurations are called transfer windows. For example, the transfer windows for Earth-Mars are open roughly once every two years. One of the goals here will be to calculate charts for choosing optimal departure times.

Problem 5: Asteroid Intercept Mission Proposal. As of today, there are close to 800 000 asteroids known in the Solar System. Many of them belong to a NEA (Near Earth Asteroid) class. The goal of this problem is to review existing known NEA asteroids, and assess the difficulty of reaching them. Next, pick one or several as target candidates and then calculate necessary maneuvers needed for a probe to leave LEO and intercept the target as it flys by close to the Earth. This problem brings in additional complexity of reaching Earth escape velocity, changing frame of reference to heliocentric, changing inclination and other orbital parameters to match those of the target.

Problem 6: Navigating with low force engine cubesat. The economic reality implies that Poland is currently incapable of launching any satellites larger than cubesats. This form factor is too small to have any conventional chemical propulsion. However, there are several possible alternative propulsion mechanisms that can be taken into consideration. One of them is a solar sail that uses solar radiation pressure to generate small, but constant acceleration. This has already been demonstrated with PW-Sat and PW-Sat 2. Another, more ambitious one is an ion engine. It uses high electromagnetic field to accelerate ionized noble gas (e.g. Xenon). The characteristic of ion propulsion is low thrust, long burn durations and high specific impulse.

A solar sail could possibly be used to perform some maneuvers, such are raising perigee and apogee of the orbit. One complication is that the force vertor always points directly outwards from the Sun. This would imply the sail would have to change orientation in various sections of its orbit around Earth. That, however, should be doable with magnetorquer, a clever mechanism that generates magnetic dipole that interfaces with Earth's magnetic field, thus providing torque and eventually rotating the spacecraft.

The goal of this problem is to propose a CubeSat mission that would use a solar sail for actual navigation. In a sense, such a mission would be a follow-up to PW-Sat 2 mission that proved that small solar sails can be deployed and used in space.

Problem 7. Website for observing Polish satellites. This requires a prediction of future fly-overs of several satellites, calculating their ground track and choosing those close passes that are close enough to specific observer positions. An additional difficulty is to take into account whether the satellite is in Earth's shadow or not. The most favourable conditions are when the observer is in the shadow, but the satellite it still in lit.

1.3. History of orbital mechanics

1.3.1. Antiquity

The history of astrodynamics (*pol.* historia astrodynamiki) is as old as the history of mankind. Our earliest ancestors were intrigued by celestial sphere since its earliest days. The oldest successful attempts to record observed sky phenomena are carvings in ancient caves many millennia ago. Although interesting from the historical and perhaps artistic point, they hold no scientific value. The first steps towards understanding the rules governing objects in the sky were taken in ancestry. Surprisingly many of the inventions and notations invented in ancient times survived and are still in use today. Babylonian Babylon (*pol.* Babilon, kultura) concept of sexagesimal system sexagesimal system (*pol.* system sześćdziesiątkowy) (a system using 60 as a base) is still in common use. The full angle is represented as six parts of 60 degrees, an hour has 60 minutes, which further splits into 60 seconds. The same is true for degrees split into minutes and then arc-seconds.

Another great example of ancient notation that withstood the trial of time is an invention of Claudius Ptolemy [19]. The Almagest, dated ~140CE, is a fascinating discourse about fundamental phenomena, such as parallax, discusses solar and lunar eclipses, a basic spherical trigonometry and much more. Surprisingly enough, one of the concepts introduced is clearly incorrect, but nevertheless is still being frequently used in modern times. Back then there were 4 planets known: Venus, Mars, Jupiter and Saturn, as those are easily visible by a naked eye. The model proposed put Earth in the middle, with translucent 7 spheres around it: 4 for known planets, one for Sun and Moon and the final seventh was reserved for the fixed, immovable stars and was considered the most perfect place. A popular saying "I'm in seventh heaven" comes from this archaic model.

Another concept that the Almagest is most known for, is the star catalog that lists over 1000 stars. Ptolemy segregated visible stars into 6 magnitudes with 1 being the brightest and 6 the faintest. This concept survived and the magnitudo scale (*pol.* magnitudo) is one of the most important scale in astronomy today. Although it has gotten a more precise definition (an object of magnitude $n+1$ is $\sqrt[5]{100}$ times fainter than n), the scale is still faithful to the fundamentals laid out by Ptolemy back in 2nd century in ancient Greece.

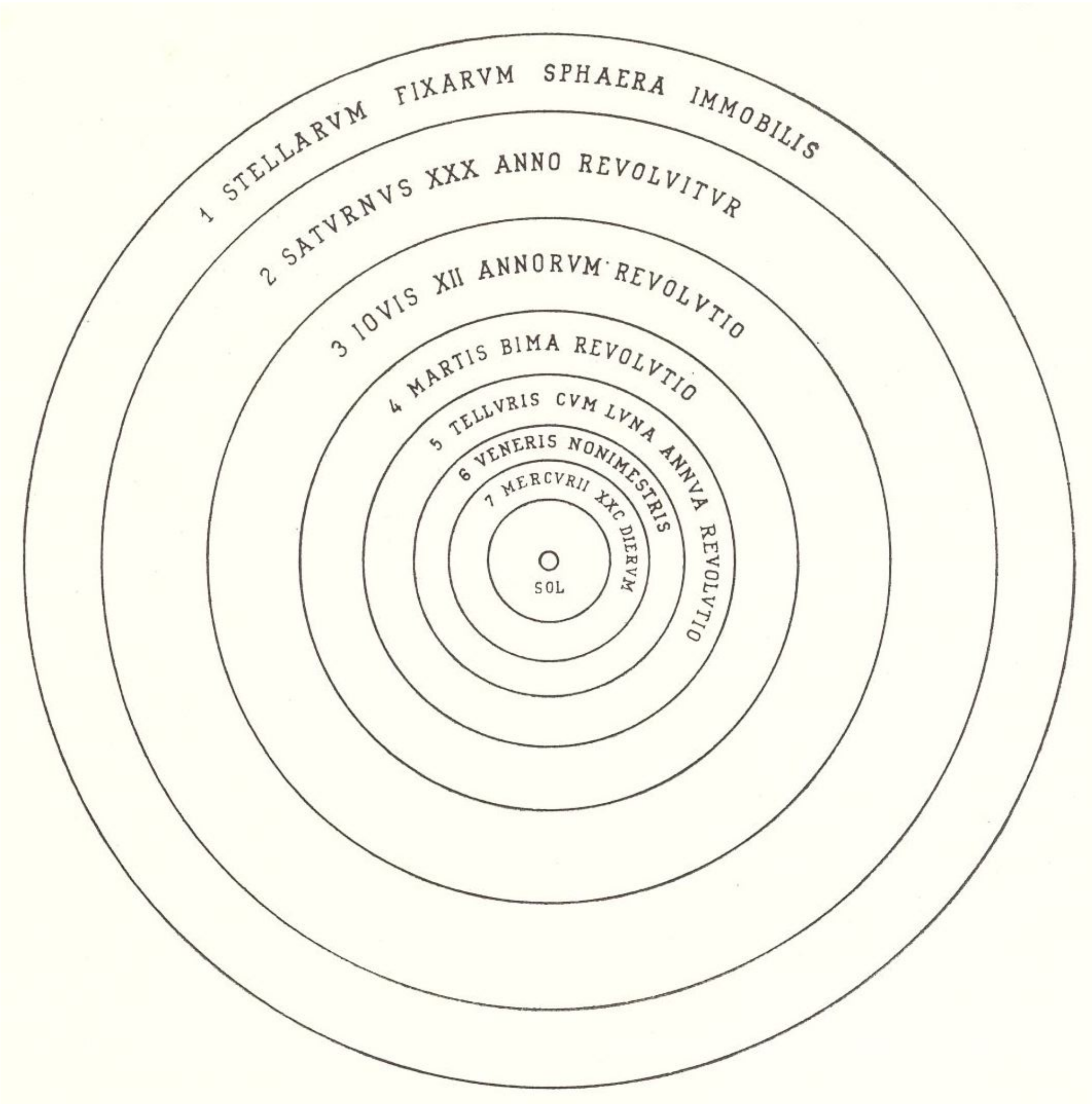

Figure 1: The first heliocentric model, proposed by Mikołaj Kopernik. This famous illustration shows both its greatest contribution and its flaw at the same time. The orbits are incorrectly assumed to be circular, not elliptical. Source: "De revolutionibus orbium coelestium" [11]

1.3.2. Middle Ages

The next major step forward in understanding heavenly bodies was made by Mikołaj Kopernik in his "De revolutionibus orbium coelestium" (latin for On the Revolutions of the Heavenly Spheres) [11], published in 1543. The proposed heliocentric model (*pol.* model heliocentryczny) was a radical change compared to prior dominating geocentric model (*pol.* model geocentryczny). The most famous diagram presenting something we call Solar system today comes from its Book I and is presented in Fig. 1 above. An observant reader will immediately notice that this same figure presents the biggest flaw of

Copernican model that caused much trouble for its author and almost led to its rejection – Copernicus incorrectly assumed that the orbits are circular. As we known today, the orbits are ellipses, but many brighter planets have low eccentricity, so their orbits are almost circular. However, the discrepancies between circular model and actual planets movements were considered a serious argument against copernican model back when it was proposed.

Another frequent misconception is that Copernicus was the first one to propose heliocentric model. Technically, the first book that discussed the concept was Narratio Prima [13], which was published in 1539, four years before ”On the Revolutions...”. However, its author Jerzy Joachim Retyk was a friend of Copernicus and was merely reporting on unpublished works of Copernicus.

The next improvement came roughly 7 decades later with “Harmonices Mundi” (latin for Harmonies of the World) [10], published in 1619 by Johannes Kelper. It defines laws of orbital motion that are considered fundamental for the modern day understanding of orbital mechanics. The major novelty was the realization that orbits are ellipses (first law), not circles. The laws also defined the change of velocity of orbiting (second law) and relation between orbital periods (third law). Another significant contribution was introduction of the concept of inertia. The advances in understanding of the universe caused some disturbance. Book I of related “Epitome of Copernican Astronomy” was listed on an index of books forbidden by an inquisition.

Moving forward, the next major step forward came with the book of “Philosophiæ Naturalis Principia Mathematica” (latin for Mathematical Principles of Natural Philosophy), by Isaac Newton [16]. Better known as “Principia”, the book is considered the most important works in the history of science. It introduced the Newton’s laws of motion, which laid foundations for classical mechanics, law of universal gravitation and derived Kepler’s laws of orbital motion (which Kepler obtained empirically). In the orbital mechanics context, it’s worth pointing out that Newton was also the first person who calculated the escape velocity for Earth [26].

1.3.3. Age of Experiments

Up until beginning of 20th century, the study of orbital mechanics was a purely observational field. This has been forever changed with the concepts of new propulsion working on the basis of expelling matter that we know as rockets today. Konstantin Ciołkowski and his “Exploring world spaces with rocket devices” [5] published in 1903 introduced not only the rocket equation that is a foundation of the modern rocket industry, but also explained how to use it to conquer space and gave many great practical ideas, such as aerodynamic tunnel. This is even more impressive, given that the book was published before the first powered flight of Wright brothers.

A significant progress has been made in Germany during World War 2. The V-2 rocket was sadly used mostly as weapon that delivered deadly explosives. However, it was significantly expanded humanity’s understanding of powered suborbital flight. While great majority of flights were configured for horizontal range with the intention of reaching distant targets such as London, V-2 could have been launched vertically. On 20 June 1944, a MW 18014, a V-2 rocket variant reached an altitude of 176 km. At that time the Kármán line was no defined yet, but nevertheless it became the first man made object that crossed the boundary of space.

After war, Werhner von Braun continued his work in United States, where he headed the space program that culminated in first manned Moon landing. Von Braun was also interested in expanding the scope of applications of rockets. In 1949, he published “Project Mars, a technical tale” [23], which is considered a first realistic proposal for interplanetary trajectories.

The last major contribution that should be noted was done by Buzz Aldrin. While his most known feat was being the second man to ever set foot on the Moon, his another substantial contribution was his doctoral thesis “Line-of-Sight Guidance Techniques for Manned Orbital Rendezvous” [3], published in 1963. It laid out the principles of precise orbital maneuvers needed for two spacecraft to meet.

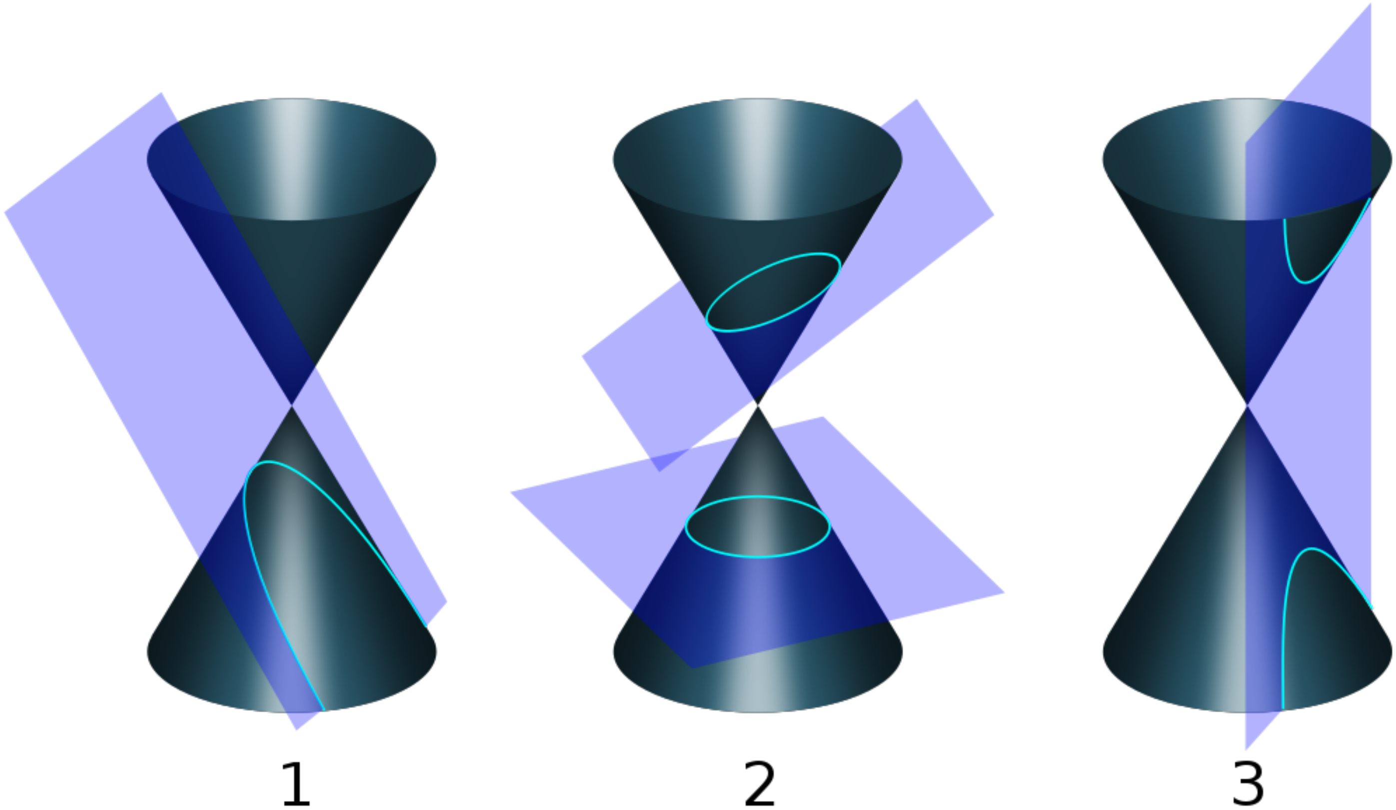

Figure 2: Orbit shapes are so called conic sections, i.e. the shapes created by intersecting a cone with a plane. Depending on the angle, the shape could be 1) parabola, 2) circle or ellipse, 3) hyperbola. Source: wikipedia.org

1.4. Orbital elements

In a general case, an orbit is one of the conic curves, i.e. one of the curves obtained as the intersection of the plane and the cone. In particular, this could be a circle, ellipse, hyperbola or parabola. Visual representation of those shapes are presented in Fig. 2 below.

Regardless of its possible varied shapes, five parameters are needed to define a shape of an orbit. Sixth parameter is needed if a location of an object along that orbit has to be specified. Those six are commonly referred to as orbital elements orbital elements (*pol.* elementy orbitalne) or keplerian elements (*pol.* elementy keplerowskie). Those parameter are discussed in detail in following paragraphs.

Many additional parameters may be used to specify optional data, such as time when the orbit was specified or describe how an orbit changes over time or is affected by various phenomena. The activity of determining why actual satellite's orbit differs from the mathematical orbit is called orbital perturbation analysis.

The five base orbital elements are:

Eccentricity (*pol.* mimośród) — This single parameter is denoted with e. Its value changes from 0 (perfectly circular orbit) to infinity. It is defined as:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} \tag{1.1}$$

where a and b are major semi-axis and minor semi-axis. Those are well defined and easily understood for circles and ellipses, but not for parabola and hyperbola. Value 0 defines a circle, values in range (0..1) define an ellipse. Those two are commonly called closed orbits as the bodies on such orbits can, given absence of external forces, repeat their flight indefinitely. Those orbit types are used to describe almost all satellites, moons, planets, asteroids and majority of comets.

Third orbit type is a parabola and it is defined for $e = 1$. When moving away from the main body, it is called escape orbit. When moving towards the main body, it's called capture orbit. That is mostly a theoretical concept, as it's impossible

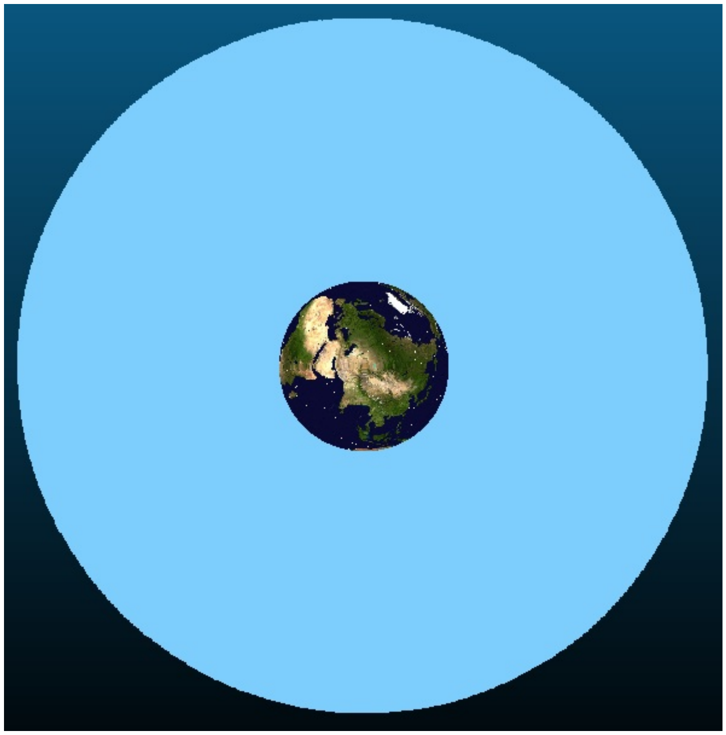

Figure 3: This image presents typical GPS orbit which has very low eccentricity (0.005) and is almost circular. Screen capture done in CloudCompare software.

to achieve eccentricity of exactly 1. Usually it's slightly less than 1 (orbit becomes highly elliptic) or slightly over 1 (orbit becomes hyperbolic).

Fourth type of orbits ($e > 1$) is a hyperbolic orbit (*pol.* orbita hiperboliczna) . Similar to parabola, those are open orbits and can be flown only once. As such, those are sometimes referenced to as trajectories, not called orbits, but trajectories. Hyperbolic orbits are used to describe planetary flybys, gravitational slingshots and objects visiting from interstellar space. One natural example of this class is 'Oumuamua, an asteroid discovered in 2017 with an eccentricity $e > 1.2$. This is the first confirmed object of interstellar origin. [18]

Major semi-axis (*pol.* półoś wielka) — An ellipse has two symmetry axes – longer (major) and shorter (minor). Denoted with a, a major semi-axis defines half of the longer axis of an ellipse. As an interesting observation, it's worth noting that an ellipse can be defined with any 2 out of 3 parameters: a, b, e. Given any two, the third one can be calculated. For orbital calculations the eccentricity is crucial, so only one semi-axis in needed and by convention a is used. Thus b is almost never used in orbital calculations.

This parameter is often imprecisely called semi-major axis. The eccentricity and major semi-axis are the only parameters needed to describe a shape of an orbit. In certain limited cases where only shape is important, but not its location, an orbit may be described in with just a and e. For example, when discussing comets or asteroids, we can classify them, calculate minimal and maximum Sun distance and roughly assess whether an object could cross Earth orbit.

Inclination (*pol.* inklinacja) – A body moving on an orbit moves on so called orbital plane. Each central body being orbited, such as Earth, has an equator. An equator extended to infinity forms an equatorial plane. An angle between equatorial plane and orbital plane is called inclination. It is typically denoted with i. This has been presented in Fig. 6. While in practice a great majority of satellites and man-made objects in space are on orbits with inclination between 0 and 90°, technically an orbit could have an inclination up to 180°. However, it's very inefficient, thus not used. Orbits that have 0 and 180°are located on the equatorial plane and the object would move over the equator. All geostationary satellites are using inclination 0. Orbits with $i \in 0..90°$ are said to be prograde (moving in the same direction as Earth surface). The orbits with $i \in 0..180°$ are said to be retrograde (moving in the opposite direction as equator). For $i = 90$ the orbit is called polar (*pol.* orbita polarna), because the satellite will pass over North and South poles. Earth monitoring satellites often use

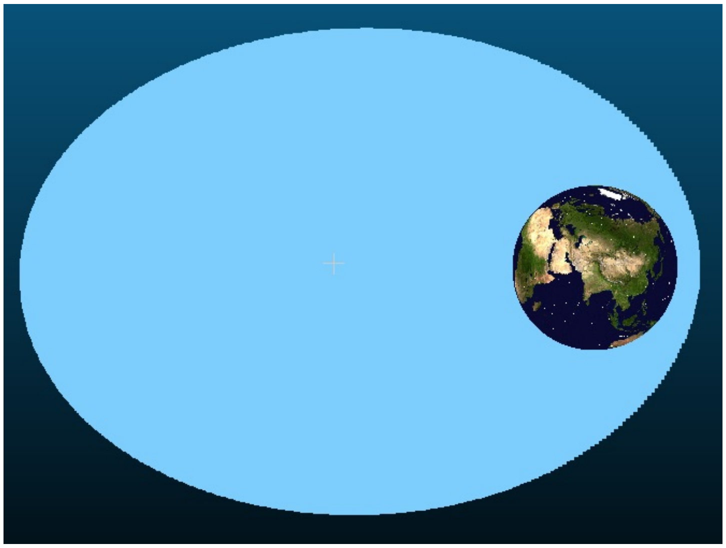

Figure 4: For very low e values the concept of eccentricity is difficult to present as the orbits look almost ideally circular. This image was created using a modified GPS almanac ($e = 0.7$) and CloudCompare software.

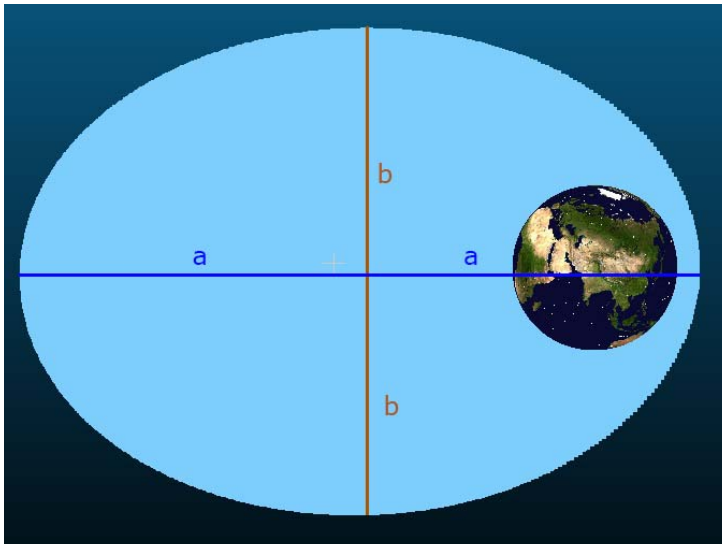

Figure 5: Major (a) and minor (b) semi-axes.

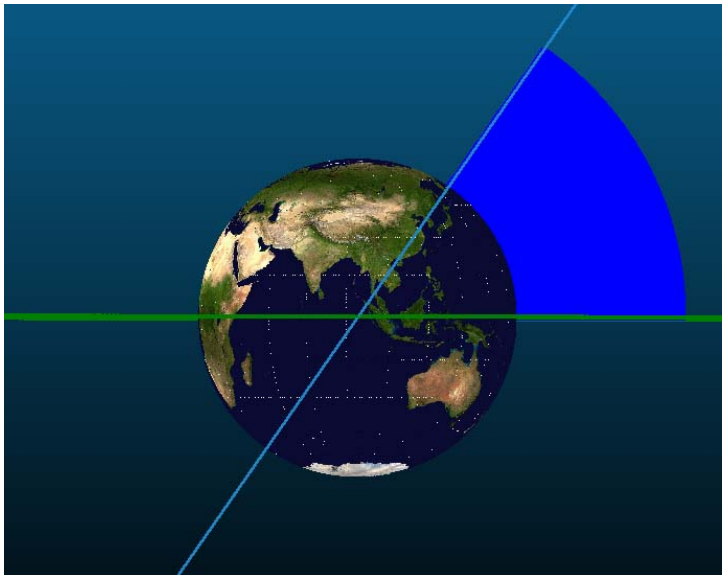

Figure 6: Orbital plane (blue line) forms certain angle with equatorial plane (green). The inclination is the angle measured from equatorial to orbital plane.

polar or near polar orbits. Retrograde orbits ($i > 90°$) are rarely used.

Most natural objects, such as planets and asteroids were created from accretion disk when the Solar system was forming. While the dust matter collapsed into large bodies, the angular momentum was mostly retained. As a result, most natural objects are rotating around the Sun in roughly the same direction. However, there are some exceptions. Triton, the largest Neptune's moon, and Phoebe, one of many Saturn's moons, are on retrograde orbits. The leading theory explaining this oddity proposes that those were not formed in their current place, but rather were formed elsewhere and then captured by their planets.

There are even fewer man-made objects using retrograde orbits, due to significant requirements for Δv to achieve those orbits and associated increased fuel and mass requirements. One example is a series of Ofeq satellites launched from Israel. Due to political stress in the region, it was unacceptable to launch rockets in eastern direction, as it could easily be misinterpreted as military attack and could trigger a war. Launching westwards (over Mediterranean sea) was a better choice.

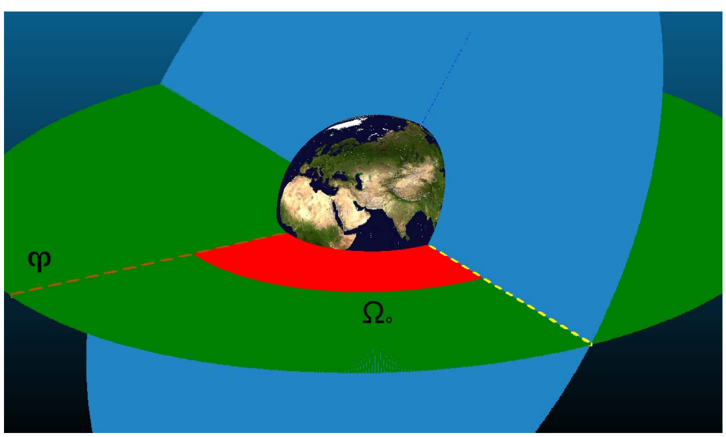

Figure 7: Aries point (Υ) and right ascension of the ascending node (Ω_0)

Right Ascension of ascending node (*pol.* rektascensja węzła wstępującego), also called longitude of the ascending node, or simply RAAN, and is often designated with Ω_0. There are several additional points and angles that have to be defined before RAAN can be explained. First, a distinguished point is needed. For Earth this was agreed to be a prime meridian, a meridian crossing Greenwich observatory in London. However, since Earth rotates, using prime meridian as a rotating frame reference would complicate calculations tremendously. A different point was selected called Aries point (*pol.* punkt Barana).

Body	Periapis (eng/pol)	Apoapis (eng/pol)
Earth	perigee/perygeum	apogee/apogeum
Moon	perilune or perycynthion/perylune	apocynthion/apolune
Sun	perihelion/peryhelium	aphelion/aphelium
Jupiter	peryjove/peryjowium	apojove/apjowium

Table 1: Common apsis names in English and Polish

The Earth completes an orbit around the Sun every year. As observed from Earth, the Sun's apparent movement completes a full circle across the background of stars. That path is called an ecliptic (*pol.* ekliptyka). By definition, it is coplanar with Earth's orbit around the Sun.

Earth's rotation is tilted roughly 23°27' and thus the equatorial plane and ecliptic plane create exactly that angle. During its precession throughout the year, the Sun spends 6 months over northern and remaining 6 months over southern hemisphere. The place an object (Sun in this example) passes equatorial plane from southern to northern hemisphere is called ascending node (*pol.* węzeł wstępujący). The opposite transition (from northern to southern hemisphere) is called descending node (*pol.* węzeł zstępujący). Technically, Aries point is an ascending node of the Sun. More intuitively, this is a point where Sun is observed on the sky during vernal or spring equinox(*pol.* równonoc wiosenna). Surprisingly enough, although the point took its name from Aries constellation (*pol.* Baran), it is currently located in Pisces (*pol.* Ryby) due to Earth precession. The orbital plane crossing equatorial plane forms two points: ascending node (where an object passes from South to North) and descending node (where an object passes in the opposite direction). The right ascension of the ascending node is defined as an angle between Aries point and the ascending node as measured on the equatorial plane. Although it defines an angle, and thus could be represented in radians ($0..2\pi$) or degrees ($0..360°$), by convention it is specified in hours minutes and seconds. Using h:m:s notation greatly simplifies astronomical calculations, e.g. when calculating raising, culmination and setting times.

Argument of periapsis (ω) (*pol.* Argument perygeum). Each orbit has its closest point to the body being orbited called periapsis and closed orbits ($e < 1$) also have a point of farthest distance called apoapsis. Apoapsis and periapsis are general terms and can be applied to any body being orbited. However, some bodies frequently used have their own suffixes. For geocentric orbits it is –gee, for lunar it is –lune, for Mars it is –areion, for Jupiter it is –jove. More commonly used names are listed in Table 1 below.

Therefore when dealing with objects around Earth, this parameter is usually referenced to as argument of perigee, although it should be kept in memory that the name is Earth specific and should not be used for objects orbiting other bodies. The argument of periapsis defines an angle on orbital plane between ascending node and a periapsis (perigee for Earth) and is typically denoted with ω. Expressed in degrees ($0..360°$) or radians ($0..2\pi$). Graphical interpretation is presented in Fig. 8 below (see angle in orange).

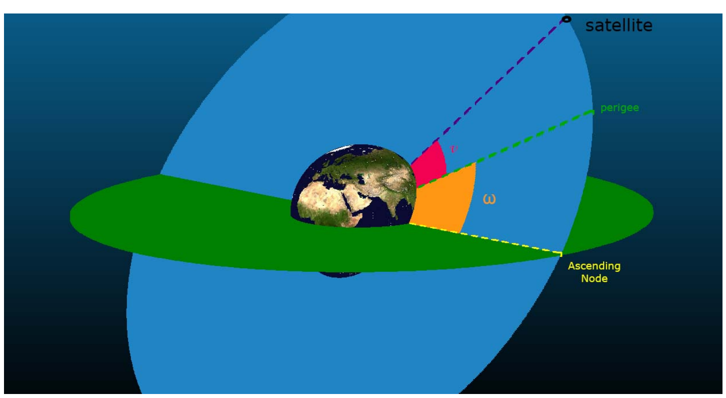

Figure 8: argument of periapsis (ω) and true anomaly (ν)

True anomaly (*pol.* anomalia prawdziwa) is usually denoted with ν (Greek letter nu), Θ or f. Previous parameters (major semi-axis a and eccentricity e) defined a shape of an orbit. The next two parameters (inclination i and raan Ω_0) defined orbital plane, i.e. the plane the orbit and the moving body is located on. With argument of periapsis (ω), it precisely defines an orbit in 3D space, i.e. a trajectory of an object moving through space. To locate the object on this trajectory, a sixth parameter is needed. The true anomaly is an angular parameter that defines the position of a body moving along an orbit. It is the angle measured on orbital plane between the direction of periapsis and the current position of a body. It is presented in Fig. 8 above, in light red color.

It is worth pointing out that second Kepler's law dictates that a line connecting a body moving on an orbit of a central body sweeps out equal areas during equal intervals of time. The most profound implication of this fact is that orbiting bodies change their linear and angular velocity on non-circular orbits as they progress throughout the orbit. Therefore it's often inconvenient to specify true anomaly and mean anomaly is provided instead. Mean anomaly (pol. anomalia średnia) is the fraction of orbit's period that has elapsed since the moving body passed periapsis and is expressed as an angle. It defines an angular distance the body would have moved if it was on a circular orbit with the same period as the actual orbit.

Another related parameter is eccentric anomaly (*pol.* anomalia ekscentyczna), which is somewhat tricky to explain. First, we need to define an auxiliary circle of radius a (*pol.* koło opisane), which is an outer bound of the orbit that touches the orbit in its periapsis and apoapsis points. The eccentric anomaly defines an angle between lines connecting periapsis with orbit focal point and a line that is perpendicular to the major axis and connecting to a point that passes through a given point P and lies on the outer circle.

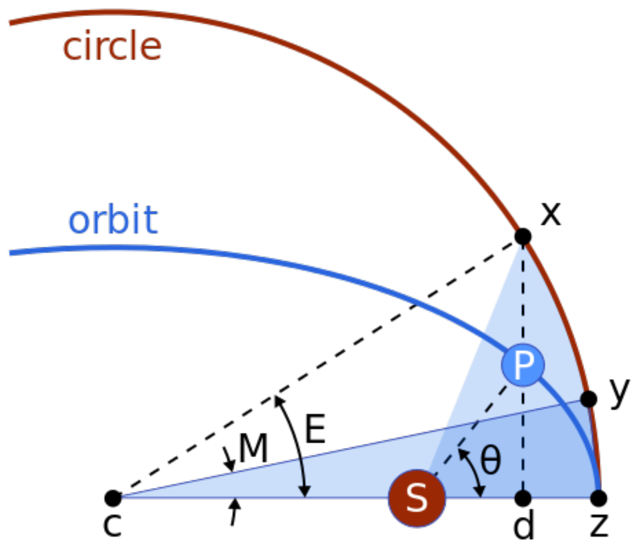

Figure 9: Eccentric anomaly E is an angle between d (a point on line of apsides) and a point x lying on both the auxiliary circle (brown) and a line perpendicular to line of apsides (d-x). source: wikipedia.org

It is sufficient to specify one of the anomalies as others can be calculated from the Kepler equation:

$$M = E - e \cdot sin(E) \tag{1.2}$$

This equation doesn't have a closed-form solution (a solution that can be obtained in a finite number of operations). Therefore it is usually calculated using numeric methods (e.g. iteratively). True anomaly ν can be calculated from eccentric anomaly E using the following equation:

$$cos\nu = \frac{cos(E) - e}{1 - e \cdot cos(E)} \tag{1.3}$$

It's possible to calculate the true anomaly from mean anomaly M, but it is more complex as it is based on Fourier expansion:

$$\nu = M + (2e - \frac{1}{4}e^3)sin(M) + \frac{5}{4}e^2 sin(2M) + \frac{13}{12}e^3 sin(3M) + O(e^4) \quad (1.4)$$

Where O(x) is a Big-O notation that indicates the omitted terms are all of order e^4 or smaller.

1.5. Orbit classification by shape

Circular orbits (*pol.* orbita kołowa) has no eccentricity ($e = 0$) and do not have designated periapsis or apoapsis. As such, the velocity and altitude are constant over time. The invariant altitude makes them well suited for Earth observation tasks. A sensor on-board, such as optical camera, or synthetic aperture radar can be designed and optimized for specific imaging distance. Circular orbits are also used in cases where the velocity variance is desired to be as low as possible. One notable example are GNSS systems. It is worth pointing out that in practice there are no perfectly circular orbits, just elliptical ones with very low eccentricity.

Elliptical orbit (*pol.* orbita eliptyczna) has eccentricity lower than one ($e \in< 0, 1)$ are by far the most common for both natural and man-made objects. Elliptical orbits have periapsis (a lowest altitude or closest approach point, where the velocity is highest) and apoapsis (a highest altitude or farthest point, where the velocity is lowest). This type or orbit is the easiest to achieve as there is no need to conduct circularization burn when inserting an object into orbit with a rocket. The elliptical orbits are also used as intermediate stages between other orbits. For example, a Hohmann transfer from circular LEO to GEO (also circular) has an intermediate step of elliptical orbit with periapsis at LEO and apoapsis at GEO altitute. The object on eliptical orbit chances its velocity over time. This is sometimes useful. For example, it's possible to design an orbit in such a way that the satellite spends more time over certain regions. A good example could by Molnyia orbits, which Russians used for communication satellites. They were optimized to spend as much time as possible over Russian territory.

Parabolic orbit (*pol.* orbita paraboliczna) has eccentricity of exactly one ($e = 1$) and is a border between closed orbits (circular and eliptical) and open orbits (hyperbolic). Parabolic trajectory is the minimum energy trajectory that allows escaping a system. When leaving a system, the orbit is called escape orbit (*pol.* orbita ucieczkowa). When arriving a system, it is called capture orbit (*pol.* orbita przechwytująca). This kind of orbit does not exist in practice, as the energy is always slightly larger or slightly smaller than this theoretical boundary. One curious interpretation of the parabolic trajectory is that its total energy is zero, which is sometimes expressed as $C3 = 0$. The interpretation for this is that the object moving away from the body is slowing down, but the gravity is getting weaker with at the pace of r^2. The departing object would stop at infinity. Obviously, that is a purely theoretical model that assumes absence of any other bodies.

Hyperbolic orbit has eccentricity of more than one ($e > 1$) and, together with parabolic, is an example of open trajectory. The naming convention of escape and capture orbits can be applied to hyperbolics, too. This type is used when departing a system (such as leaving Earth vicinity) or when visiting a system (such as Jupiter fly-by). There are few natural bodies that move along such trajectories, but they are known to exist. Two most famous hyperbolic objects are 'Oumuamua and I2/Borisov. Both are proven to be interstellar visitors to the Solar System. (*pol.* kometa!I2/Borisov).

Radial trajectory (*pol.* trajektoria radialna) is a special case of degenerated orbit. The orbiting body moves directly away from or towards the center of the orbited body. Such trajectories are seldom used, as they don't have practical value. However, they can be used to model certain phenomena, such as throwing something directly up or explaining why launching a rocket straight up would not achieve orbit, regardless of how powerful the rocket is.

1.6. Orbit classification by altitude

A very common classification segregates orbits depending on their altitude. This is useful nomenclature for orbits that are circular or slightly elliptical, i.e. their periapsis and apoapsis are not radically different. The **Low Earth Orbit** (*pol.* Niska orbita wokołoziemska) is typically defined as an orbit with altitude of 2000 km or lower. While the (*pol.* linia Kármána) set at 100 km is commonly considered a boundary of space, the actual atmosphere extends far beyond this line. In general, orbits with periapsis lower than 185 km are considered unstable. As such, a practical approach is to define LEO as orbits of periapsis as low as 185 km and apoapsis no greater than 2000 km. A great majority of satellites are in LEO orbits. This is particularly popular orbit for Earth Observation satellites. Since this is the easiest orbit to achieve, most missions with low energy budgets use this type of orbits. The orbital period is typically many times per day. For example, the International Space Station (ISS) is using 408 x 410km orbit and has an orbital period of 92 minutes. This translates to rougly 15,5 orbits per day.

The next category is MEO or **Medium Earth orbit** (*pol.* średnia orbita wokołoziemska). It is typically defined as an orbit with altitude over 2000 km and less than 35 786 km. This orbit is frequently used by GNSS systems. In particular, orbits of altitude of 20200 km are popular, as the orbital period for such an altitude is 12 hours. This is useful for getting a repeating daily patterns.

There is one distinct altitude of 35 786 km that defines a separate class of orbits. For this altitude the orbital period matches sidereal day, which is 23h 56m 04s. Any orbit that has this altitude will appear as not moving eastwards or westwards as observed from the Earth surface. Such class of orbits is called **geosynchronous orbit** (*pol.* orbita geosynchroniczna) and are often abbreviated as GSO. GSO with a non-zero inclination will appear as moving roughly around the north-south line. A special case of GSO with zero inclination is called **geostationary orbit** (*pol.* orbita geostacjonarna), often designated as GEO. It has a major advantage to rotate at the same pace as Earth and not move in the north-south direction. As such, GEO satellites appear stationary as observed from Earth. This brings a major advantage of being able to use fixed (non-tracking) directional antennas. Once set up, they can maintain good reception for a long time, assuming the orbital drift is negligible. This orbit is very popular with communication satellites, in particular satellite TV, radio and other communication. There are so many satellites in GEO orbit that they form a ring around Earth. Since the plane and altitude is pre-determined for GEO satellites, there is a simplified notation that describes object's location. It uses a longitude of the sub-satellite point. For example, a Hot Bird 10 satellite is located at 33°East.

Due to orbital drift, satellites in GEO need to conduct small correction maneuvers to remain in GEO orbit. The amount of remaining fuel is the typical limiting factor for the operational lifetime of GEO satellites. Once a satellite approaches the end of its lifetime, it is moved away from GEO to not clutter this precious space.

Finally, orbits above GEO are considered **High Earth orbit** (*pol.* wysoka orbita okołoziemska). Such orbits are used infrequently, as the energy requirements are even larger than for GEO. The HEO orbits are used for missions that require considerable distance from Earth. There are few satellites in HEO orbits. Most of them a deep space observatories, especially those affected by Earth's magnetosphere.

The preceding paragraphs described most typical orbits around Earth. Similar classification can be used for other bodies. For example, NASA uses LLO or Low Lunar Orbit to designate orbits around the Moon with altitude lower than 100km. Such low orbits on the Moon are possible, because it has almost no atmosphere and there's negligible friction while flying at that altitude.

1.7. Orbit classification by inclination

Orbits are often classified or characterised by other parameters. One of the particularly important one is inclination. An orbit that is on the equatorial plane has zero inclination and is called **equatorial orbit** (*pol.* orbita równikowa). . Geostationary orbit is an equatorial orbit with an altitude of 35 786 km. Orbits that have a high inclination, i.e. close to 90°are passing at or near the poles. Such an orbit is called **polar orbit** (*pol.* orbita polarna) . This type of orbits is particularly well suited for Earth Observation, as the satellite passes over almost all area. In general, the satellite can pass over land area up to the latitude of its inclination. For example, for a satellite to cover whole Poland area, it needs to be placed on an orbit with an inclination of at least 54°50' (the longitude of the northernmost point in Poland – Jastrzębia Góra).

Objects that circumvent in the same direction as Earth rotation have inclination between 0 and 90°are said to be on **prograde orbit** (*pol.* orbita prograde) . Objects that cirvumvent in the opposite direction as Earth's rotation have inclination between 90 and 180°are said to be in the **retrograde orbit** (*pol.* orbita retrograde) . The prograde/retrograde naming convention is also convenient to describe orbital maneuvers. The prograde direction means in the direction of movement (”accelerating forward”), while retrograde means the opposite direction (”breaking”).

Great majority of satellites are using prograde orbits as launching eastwards decreases the Δv requirements. Also, objects moving in prograde direction appear to move slower when observed from the surface, so most activities, such as imaging or communication, is easier. However, if the mission has special requirements, a retrograde orbit provides some unique benefits.

Real orbits drift over time due to small external factors. This is discussed in more detail in 1.14. For the time being, it is worth pointing out that there are two special values of inclinations. The first one in 98°, which causes the orbital plane to rotate slowly over time at the rate of exactly one revolution per year. This is useful for Sun-synchronous orbits, to be discussed in the following section. Another special value of inclination is 63.4°, which causes the orbit to not drift at all. This value is called **critical inclination** (*pol.* inklinacja krytyczna) and is used by Molniya and Tundra orbits, also to be discussed in the next section.

1.8. Special purpose orbits

There are several uncommon or special purpose orbits. While they all fall into the above classifications, they have certain properties that allow the satellites to achieve their mission goals better. They’re listed roughly in the order or popularity.

Many Earth Observation missions can benefit from observing the same area at the same local solar time, e.g. an air quality mission would be interested in obtaining measurements that are consistently on the same time of the day, so impact of external patterns, such as industry operation througout the day or people heating their homes with stoves in the evenings, is minimized. To achieve this goal, it is necessary to take advantage of perturbations. This phenomena is described in more detail in 1.14. Briefly, certain external factors, such as Earth not being perfectly round or solar radiation pressure, cause the spacecraft’s orbit to slightly change over time. This impact is small, but measurable. To achieve the goal of fly-over at the constant time, The orbit has to precede slowly. The ascending node is expected to conduct a full circle over the year, at exactly the same rate as Sun’s apparent movement over the sky. Such an orbit is called **Sun-synchronous** (*pol.* orbita heliosynchroniczna) and is often abbreviated as SSO.

A Sun-synchronous orbit can be chosen to always fly in the dawn/dusk. The Sun is almost always visible, which is of particular importance for Sun observation missions.

Interestingly enough, the Sun-synchronous orbit requires perturbations strong enough to provide rotating momentum equal to one full revolution per year. The major part of such perturbations in case of Earth comes from its oblateness. It is possible to construct Sun-synchronous orbits around Mars, because it’s also oblate, but not Venus, which is almost perfectly spherical [30].

Another type of special orbits are **Molniya** and **Tundra orbits**. . They take the name from Russian series of communication satellites. Both orbits are trying to solve the problem of radio communication in high latitude areas, such as northern parts of Russia. The orbits take advantage of several factors. First, they use critical inclination ($i = 63.4°$), so they have a zero drift over time (i.e. the orbit does not shift eastwards or westwards). The next aspect here is that they are highly eccentric ($e \approx 0.74$, which results in a significant difference in periapsis (6650 km) and apoapsis (46550 km). The velocity is also radically different. As a result, the spacecraft spends disproportionally large part of its orbital period near its apoapsis. Together with the critical altitude, the apoapsis always falls over the same chosen area of the Earth. As a result, each of Molniya satellites had 8 operational hours out of its 12h period. A constellation of only 3 satellites in principle provided permanent converage over desired area of northern Russia.

The Tundra orbit uses very similar approach, but has a period of full sidereal day (23h 56m 4s), instead of half sidereal day as Molniya does.

1.9. Lagrangian points and exotic orbits

All of the orbits discussed so far were solutions of a 2-body problem, i.e. they assumed a major body, such as Earth, is being orbited by a much smaller object, such as a spacecraft and any external influences, such as Sun's or Moon's gravity have only very minor impact and is modelled by pertubations. This is a good approach in case of considering relative proximity of the major body. This is often conveniently modelled as a **sphere of influence** (*pol.* sfera wpływu). However, when the distance is large enough or there is another heavy body in proximity, the classical 2-body problem is insufficient and 3-body problem has to be considered. Two most common examples are Earth-Sun and Earth-Moon systems. One particular aspect of 3-body system is that there are points where the gravitational forces of the two major bodies are equal. Depending on the configuration, the forces can reinforce each other or mostly cancel one another. Those points are called **lagrangian points** or **libration points** . Calculating libration points is out of scope for this thesis, as they involve dealing with complex differential equations with no analytical solutions and require numerical methods. Reader interested in the details is encouraged to read Chapter 2.12 of [6]. An overview of the libration points of the Earth-Moon system are presented in Fig. 10.

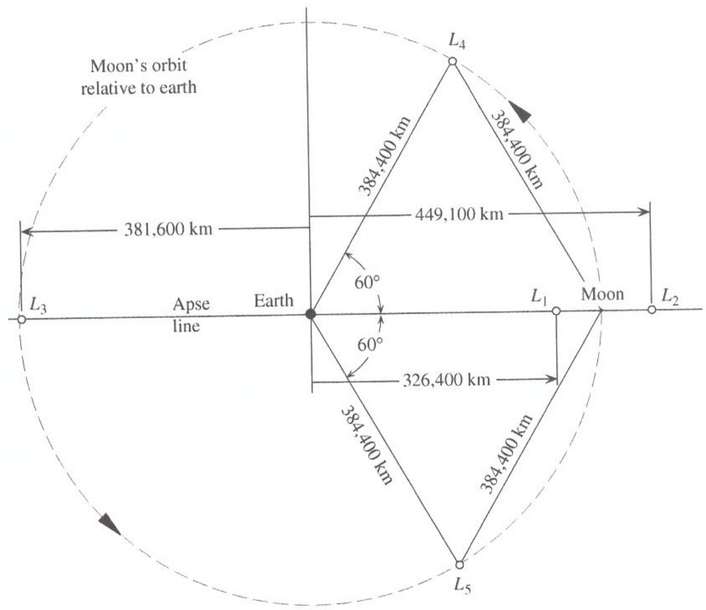

Figure 10: Any given 3-body system, such as Earth-Moon depicted here, has libration points where gravitational forces of two major bodies are equal. A small body, such as spacecraft, positioned in any lagrangian point would seem to orbit the body the observer is located at. Source: [6]

A **halo orbit** (*pol.* orbita halo) is an orbit around L_1, L_2 or L_3 Lagrange points. Although there is no mass there, the attracting forces of two other large bodies in the vicinity cause that this point can be orbited. The halo orbits are usually unstable and require stationkeeping, although the magnitude of corrective maneuvers tend to be low. The concept of halo orbit was first proposed in 1968 by [7] in his Ph.D. thesis. There are very few spacecraft that used such an orbit. Herschel Space Observatory by ESA uses 800 000 km average halo orbit around L_2 of the Sun-Earth system. The spacecraft on average remains at a distance of 1.5 million km from Earth.

A **Lissajous orbit** (*pol.* orbita Lissajous) is a quasi-periodic orbital trajectory around Lagrangian points. Its major flaw is that it is not periodic, meaning that each evolution is slightly different than the previous one. However, in return the major advantage to halo orbit is that it does not require stationkeeping maneuvers, and thus is much cheaper for maintain in the long time. In most applications the L_4 and L_5 points are considered stable. [28] claims the orbits around L_4 and L_5 points can last few millions of years. Ignoring perturbations by other planets, they can be stable for billions of years. An example of Lissajous trajectory is shown in Fig. 11. There are several spacecraft that use this kind of orbit: ACE, SOHO, DSCOVR, WMAP, Genesis, Herschel and Panck observatories, Gaia, two THEMIS spacecraft and Queqiao.

[31]

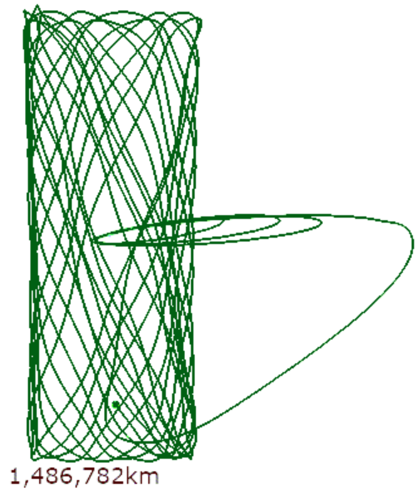

Figure 11: Lissajous orbit is quasi-stable orbit around Lagrangian points. An example of Wilkinson Microwave Anisotropy Probe orbiting around L_2 libration point of the Earth-Moon system. Source: [28]

A **Near-rectilinear Halo Orbit** (*pol.* prawie prostoliniowa orbita halo) is a class of halo orbits. It has recently gained popularity after NASA announced [14] in 2020 that NRHO orbit will be used for the Lunar Gateway, a new manned space station to be built in the general vicinity of the Moon. The Gateway is expected to be positioned around L_1 point of the Earth-Moon system. After this announcement, the orbit became of high interest to many. Advanced Space is working on CAPSTONE mission, a 12U cubesat to be launched to NRHO orbit. Three companies (Blue Origin, Dynetics and SpaceX) were picked by NASA to provide launch services and lunar lander architecture. Many variants being discussed involve rendezvous with the Gateway at NRHO orbit.

There are several attributes of NRHO that makes it attractive. It is easier to get to from Earth, compared to Low Lunar Orbit. This makes the general supply chain from Earth much more sustainable. NRHO passed over the Moon poles, which are the most attractive prospective sites for first lunar bases. Due to absence of atmosphere, any ice deposited on the Moon slowly evaporates when exposed to direct sunlight. However, there are craters near the poles, that remain in permanent shadow, which prevents the evaporation and current results strongly suggest they have rich deposits of water ice. On the other hand, the top of the crater rims are in permanent sunlight, which offers tremendous benefits. It is worth pointing out that the Moon is tidaly locked to Earth, which means that one Lunar day is 27 Earth days and 7 hours and average lunar night lasts over 13 days. The difficulty of surviving the lunar night by a prospective base is substantial.

There are several other advantages of NRHO. The orbital plan is perpendicular to Earth-Moon line, which means that objects on NRHO are never in the Moons shadow and can always maintain radio communication with Earth. Finally, the orbit has low escape velocity, which may become very useful for future Mars missions. The human capsule departing for Mars could be assembled at NRHO and be provisioned from the Moon base, in particular in the context of providing water and its products (oxygen and hydrogen).

The NRHO selected by NASA has a period of 7 days, periapsis of 3000 km and apoapsis of 70000 km. A visualization of the NRHO orbits are presented in Fig. 12. Reader interested in learning more about NRHO is encouraged to read [31].

1.10. Reference systems

Comment: Opisać powszechnie używane systemy odniesnienia:

- ECEF, Geocentric Equatorial Coords System, Heliocentric elliptic coords, Perifocal, ENU, w szczególności TEME (używany w modelach SGP4)...

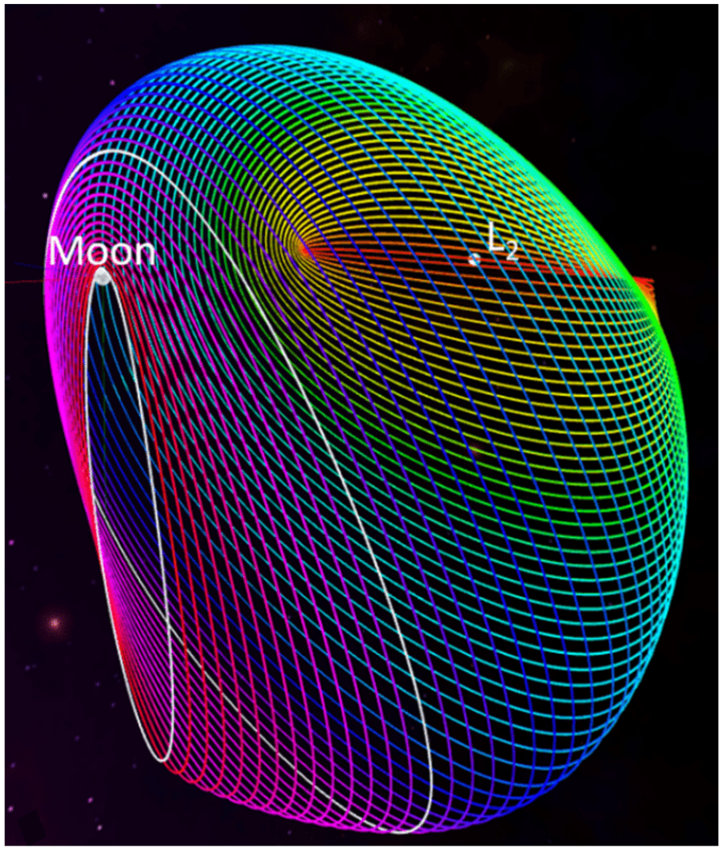

Figure 12: A family of Near Rectilinear Halo orbits around the Moon-Earth system. Source: [31]

1.11. Popular orbital notations

Comment: Opisać popularne formaty zapisu danych satelitarnych. Trafiłem na Panów artykuł "Analiza porównawcza formatów danych satelitarnych w systemie GPS". Chciałbym do niego nawiązać i trochę go rozbudować.

- describe most frequently used notations and formats: TLE, Yuma, SEM, MPCORB, including their advantages and weaknesses.

1.12. Orbital maneuvers

- Impulsive maneuvers
- Hohmann transfer
- Bi-elliptic transfer
- Phasing maneuvers - changing orbital phase
- Apse line rotation
- Chase maneuvers
- Plane change maneuvers
- Orbital Rendezvous
- Satellite avoidance maneuvers
- Non-impulsive orbital maneuvers

1.13. Interplanetary trajectories

- Interplanetary Hohmann transfers
- Gauss problem
- Sphere of influence
- Method of patched conics
- Departure/Injection burns (TLI, TMI, TVI)
- Gravity assists (Oberth effect)

1.14. Orbital perturbations

Comment: Perturbacje są niezwykle szerokiem tematem. W zależności od zakresu obiektów, które chcemy poruszyć (tylko satelity okołoziemskie, czy też może nieco szerzej), można wspomnieć o dodatkowych aspektach.)

- concept of perturbers
- atmospheric drag
- non-spherical gravitational field (Earth oblateness)
- solar radiation pressure
- lunar gravity
- solar gravity
- planets (for heliocentric orbits)
- asteroids (for heliocentric orbits)
- Yarkovsky effect -
- Yarkovsky-O'Keefe-Radzievski-Paddack effect, or YORP effect
- Poynting-Robertson drag
- Pioneer anomaly
- Orbital resonance (Secular and Lindblad resonance)

2. Related work in Astrodynamics

This chapter describes the current state of the art in the broad topic of orbital mechanics. It can be roughly split into four sections. The first one is a review of current, up to date books and monographies. The second one is an attempt to describe latest papers and articles in related fields. The third section is an overview of existing software, its capabilities and limitations. The fouth one is selection of more interesting space missions and satellites that are important or interesting from the orbital mechanics perspective.

2.1. Related books

BOOKS:

- H. Curtis - Orbital Mchanics for Engineering students
- J. Wertz - Mission Geometry; Orbit and Constellation Design and Management
- J. Wertz & W. Larson - Space Mission Analysis and Design, 3rd ed.
- P. Fortescue et al. - Spacecraft Systems Engineering
- C. Specht - System GPS
- R. Bate et al. - Fundamentals of Astrodynamics
- J. Meeus - Astronomical Algorithms

2.2. Related software packages

2.2.1. Systems Tool Kit (STK)

AGI STK - Systems Tool Kit - a powerful software developed by AGI described as multi-domain mission-level software for system design, operations, and analysis. (commercial, GUI with scripting capabilites, most powerful tool in the market, also the most expensive and closed source) https://www.agi.com/products

2.2.2. General Mission Analysis Tool (GMAT)

GMAT - General Mission Analysis Tool - a space trajectory optimization and mission analysis system developed by NASA. https://sourceforge.net/projects/gmat - GUI with scripting capabilities, closest open source competitor to STK, sometimes unstable

2.2.3. Poliastro

Poliastro - https://docs.poliastro.space/en/stable/, pure python, optimized for ease of use and contribution, less powerful than the alternatives

2.2.4. Optimum Interplanetary Trajectory Software (OITS)

2.2.5. Orekit

Orekit (https://www.orekit.org/ - Java library, Mature, widely used, not so focused on interplanetary applications, mostly Earth-centric orbit

2.2.6. Spice

Spice (https://naif.jpl.nasa.gov/naif/toolkit.html), rather old, C, Fortran, IDL, Matlab, python libraries, Battle tested collection of functions, Generating the kernels requires some expertise, used by NASA, (kernels - files that describe the mission)

2.3. Cesium

Cesium - an Earth visualization library suitable for interactive web experience.

2.4. Python libraries

2.4.1. SGP4 models

SGP4 - orbital propagation models proposed by T.S.Kelso that are currently used by NORAD and NASA (the original Pascal library and its reimplementation in Python)

2.4.2. Orbital predictor

2.4.3. tle-tools

2.4.4. AstroPy

AstroPy - astronomy library, timeframes, reference systems

2.5. Notable papers

NOTABLE PAPERS:

- A. Hibberd et al. - Sending a Spacecraft to Interstellar Comet C/2019 Q4 (Borisov), https://arxiv.org/abs/1909.06348

3. Developed Software

This chapter describes the design assumptions, architectural decisions and the actual software implemented.

TODO: Describe the proposed system. Major assumptions:

- written in python
- using industry standards (orbits defined in TLE format, using SGP4 algorithms published by NORAD)
- portable

3.1. Design assumptions

3.2. Software architecture

3.3. Installation procedure

3.4. User's Reference Manual

4. Usage scenarios

This chapter describes a number of applications and problems that were solved.

4.1. Problem 1: Georeferencing Satellite Images

With the advent of cheap software defined radio (SDR) hardware, it's possible to receive satellite VHF and UHF satellite transmissions. As part of a different assignment [20], the author designed and built a satellite ground station that is able to receive VHF transmissions in the 137MHz band. The transmissions convey Earth Observation images from NOAA satellites. The NOAA images are transmitted using APT encoding and contain current view of the atmosphere. They are imaged in two IR bands: near IR and far IR, although the NOAA satellites have several sensors and can be reconfigured to transmit photos from different bands. In both cases, the images contain a scan along the fly-over path. The visual data on its own is hard to interpret, because characterstic land features in Europe are often clouded and the image is very wide (usually covers most of the Europe). This issue can be seen in Fig. 13. The software should use image acquistion time and known orbital trajectory to provide georeferencing information. This can then be used to overlay country boundaries, geodetic grid and other types of information. The practical goal is to increase readability of images received in a working project. This task has been proposed by prof. Marek Moszyński.

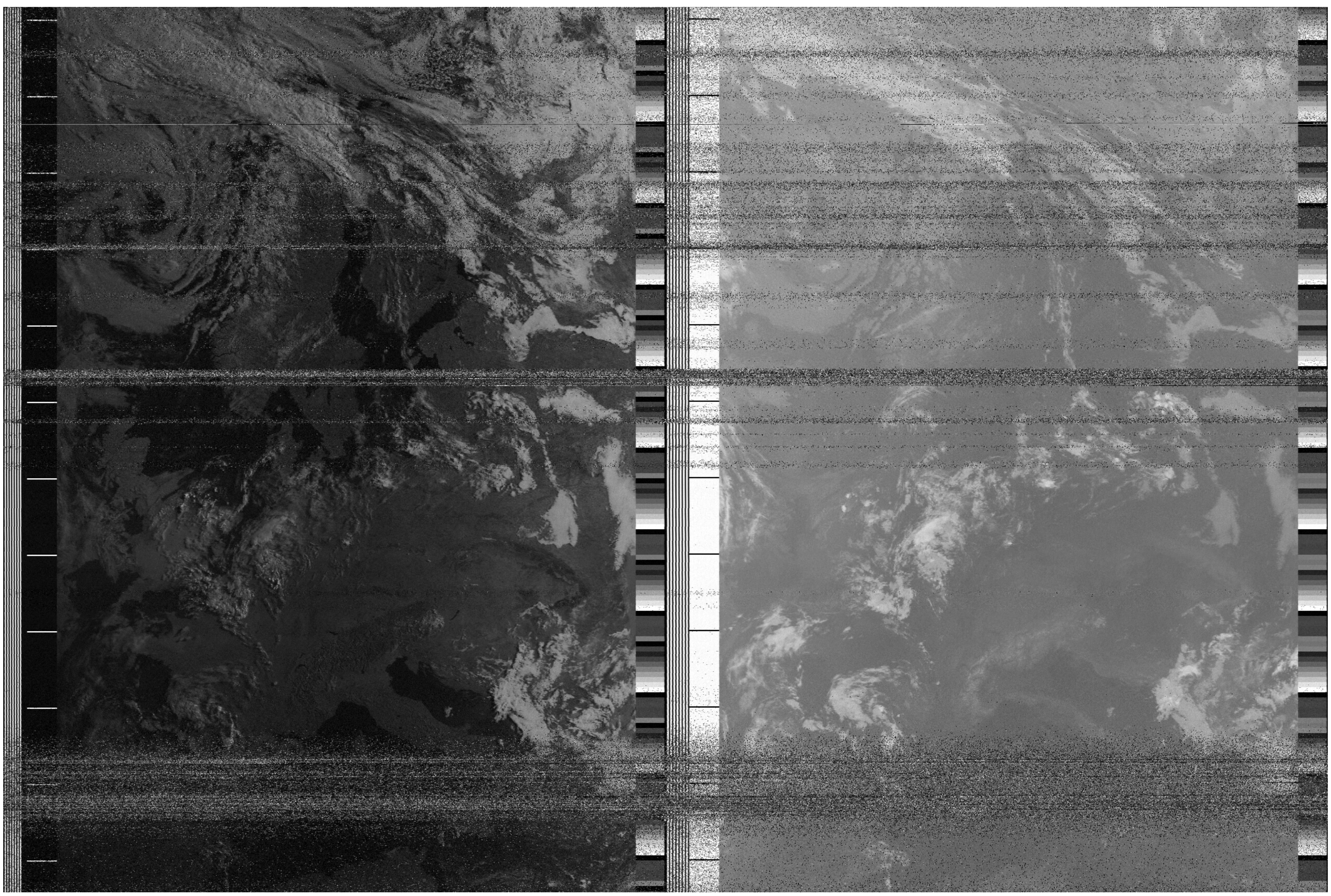

Figure 13: Image from NOAA-19 satellite, no georeferencing information. The Satnogs project recorded over 2000 observations since its inception in Jan. 2020. This particular image was chosen because of a reasonably clear weather and good reception quality. The land contours are reasonably clearly seen. This will simplify the visual verification of the borders overlay.

Ellipsoid reference	Semi-major axis a	Semi-minor axis b	Inverse flattining $1/f$
WGS 72	6 378 135.0 m	≈ 6 356 750.52 m	1/298.26
GRS 80	6 378 137.0 m	≈ 6 356 752.314 140 m	298.257 222 100
WGS 84	6 378 137.0 m	≈ 6 356 752.314 245 m	298.257 223 563

Table 2: Shape of Earth as defined in popular Geodetic Systems

4.1.1. Calculating orbital position

The georeferencing procedure requires several parameters: orbital parameters, the time of transmission start t_{LOS}, lenght of transmission Δt and geometry of the optical sensor mounted on a satellite. In a general case, two timestamps are required: time of transmission start t_{AOS} for beginning (in satellite industry called Acquisition of Signal AOS, (*pol.* AOS)) and end of the transmission time t_{LOS}. However, since NOAA satellites transmit (Loss of Signal, LOS LOS, (*pol.* LOS)) their images continuously at the rate of two lines per second, the end of transmission time can be substituted with transmission duration d, which can be either expressed in seconds or number of lines expressing height of the received image:

$$t_{LOS} = t_{AOS} + \frac{h}{2} \tag{4.1}$$

The first step is to calculate satellite position at AOS and LOS. This can be done using SGP4 models (see Section 2.4.1) and TLE orbital parameters (see 1.11). It is worth noting that the orbital parameters are not constant and evolve slowly over time. As a practical matter, it is essential to keep TLE parameters from the time an observation was recorded. The SGP4 models take TLE parameters and a timestamp expressed as Julian Date (see 1.10) and produce results in a Cartesian position and volocity versus Time Since TLE Epoch in the True Equator, Mean Equinox (TEME) coordinate system (see 1.10). Those TEME coordinates can be converted to Earth-Centered, Earth-Fixed (ECEF) reference frame. This, however, need to take into consideration the time as one frame or reference is fixed and the other is rotating. Depending on the Earth motion model, this may range from reasonably simple if only circular rotation is taken into account to suprisingly difficult if precession and nutation are also considered. The implementation author used (2.4.1) is based on AIAA paper [1].

4.1.2. Converting TEME to Geodetic coordinates

Earth is not perfectly round and due to centrifugal forces is slightly bulged at the equator and flattened at the poles. An exagerated view of the Earth cross-session is presented in Fig. 14. The Earth curvature is defined by two radii measured at different places: a and b. The proper term for a is semi-major axis, but it's really an equatorial radius. Conversely, the proper term for b is semi-minor axis, but it's a radius measured at the poles. Polar radius, b, and equatorial radius a are not equal and are bound by a parameter called flattening f.

$$b = a(1 - f) \tag{4.2}$$

As a practical convenience, the f value is a small fraction, so the value is often expressed as $1/f$ and is often referred to as inverse flattening. Many geodetic systems define this value. a, b and $1/f$ are defined by many geodetic systems. More popular ones are shown in 2 below. Surprisingly enough, the TLE format and the SGP4 models are based on WGS 72 ellipsoid refrence, as this model was used by US Departemnt of Defense. For details, see [27]. Note the difference between WGS 72 and WGS 84 is only 1.8 meters, which for the most applications (except GNSS) is acceptable.

The curvature introduces a new phenomena, which may not be obvious. For a spherical Earth the local zenith (defined as right angle from the local horizon, or more colloquially "straight up"), the observer and Earth center always form a straight line. That is not the case on oblate Earth. The distinction between the angle between equator and observer ϕ' and angle between equator and local zenith ϕ is an important aspect of the coordinates conversion algorithm.

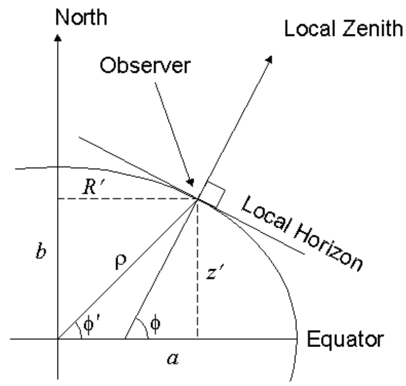

Figure 14: Oblate Earth

Assuming ECI position of the satellite to be $[x, y, z]$, the latitude on spherical Earth is

$$\phi' = tan^{-1}\left[\frac{z}{\sqrt{x^2+y^2}}\right] \tag{4.3}$$

and the longitute is:

$$\lambda_E = tan^{-1}\left[\frac{y}{x}\right] - \Theta_g \tag{4.4}$$

whre Θ_g is the Greenwich Mean Sidereal Time (GMST), expressed in radians. In a general case, the satellite altitude would be defined by

$$h = \sqrt{x^2+y^2+z^2} - R_e \tag{4.5}$$

where R_e is the Earth radius. However, with the georeferencing problem at hand, we're mostly concerned with the on-surface projection and the satellite altitude is not a concern.

For an oblate Earth, the calculation is more complex and requires several iterations. To calculate the geodetic latitude of the subsatellite point, the algorithm starts with an approximation of ϕ with ϕ' as calculated for round Earth and then goes through a series of iterations until the error is within the desired tolerance. The iteration is as follows:

$$\phi_i = \phi \tag{4.6}$$

$$C = \frac{1}{\sqrt{1 - e^2 \cdot sin^2(\phi_i)}} \tag{4.7}$$

$$\phi = tan^{-1}\left[\frac{z + aCe^2 \cdot sin(\phi_i)}{R}\right] \tag{4.8}$$

and the approximation error is $|\phi - \phi_i|$ and R is a distance of the satellity from the Earth axis of rotation and can be calculated using the $R = \sqrt{(x^2 + y^2)}$ formula. It's worth noting that the algorithm converges very quickly. T.S. Kelso [9] gives an example of Mir station calculations. The iterations gave the following errors (in degrees): first 0.180537, second 0.000574 and third 0.000002.

4.1.3. Georeferencing wide images

One significant problem with NOAA satellite images is that they cover a wide area, which are a significant portion of the sphere. The image swath is roughly 2900km and the length depends on the reception quality, but it is many thousands kms. This means that the projection of geodetic coordinates to x, y coordinates of the image cannot assume meridians or parallels to actually be parallel on the image.

Given two coordinates of ϕ_1, λ_1 and ϕ_2, λ_2, the distance and azimuth between them can be calculated using the following equations:

$$az = atan\left[\frac{sin(\Delta\lambda)}{cos(\phi_1) \cdot tan(\phi_2) - sin(\phi_2) \cdot cos(\Delta\lambda)}\right] \tag{4.9}$$

where $\Delta\lambda = \lambda_2 - \lambda_1$. Similarly, the distance can be calculated using the following formula:

$$dist = acos(sin(phi_1) \cdot sin(\phi_2) + cos(\phi_1) \cdot cos(\phi_2) * cos(\Delta\phi)) \tag{4.10}$$

Note the resulting azimuth and distance are both expressed in radians.

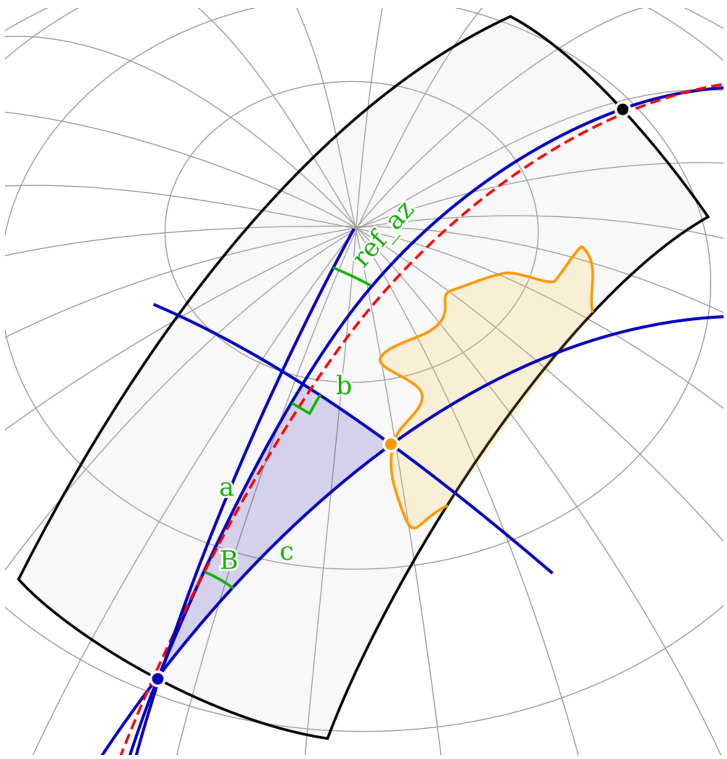

Figure 15: Projecting geodetic coordinates on the image.

The preliminary step requires to calculate the satellite azimuth. This is calculated using subsatellite points for AOS and LOS, and converting to $\phi_{AOS}, \lambda_{AOS}$ and $\phi_{LOS}, \lambda_{LOS}$ as described in the earlier sections. Then the satellite azimuth can be calculated using the equations above. This is the reference azimuth that corresponds to the column of pixels in the middle of the image. To calculate pixel position on the image, one needs to convert the ϕ, λ coordinates using the following steps:

1. calculate reference azimuth az_{ref} using $\phi_{AOS}, \lambda_{AOS}$ and $\phi_{LOS}, \lambda_{LOS}$
2. calculate azimuth az between AOS and the point being converted.
3. calculate distance c between AOS and the point being converted.
4. calculate $B = az - az_{ref}$
5. calculate a using $a = atan(cos(B) \cdot tan(c))$
6. calculate b using $b = asin(sin(B) \cdot sin(c))$
7. calculate x position using $x = -b/xres$
8. calculate y position using $y = a/yres$

where xres and yres are the image dimensions, expressed in pixels. Finally, the NOAA images are actually two images with additional control/sync data on the sides, so extra offset needs to be calculate for those. This process is demonstrated in Fig. 15. This particular algorithm is roughly based on [17].

Using the algorithm above any geodetic coordinates can be converted to x, y coordinates of the image. As a proof of concept, three types of data were overlayed. First, a set of country borders were shown in yellow. The overlay may is using SHP files from The Natural Earth service [15] that were simplified using MapShaper service [12]. Second, a geodetic grid was overlaid to better show the Earth curvature (and demonstrate the inappropriateness of rectangular projection in the process). Finally, the third type of data was the ground station location. An example result can be shown in Fig. 16.

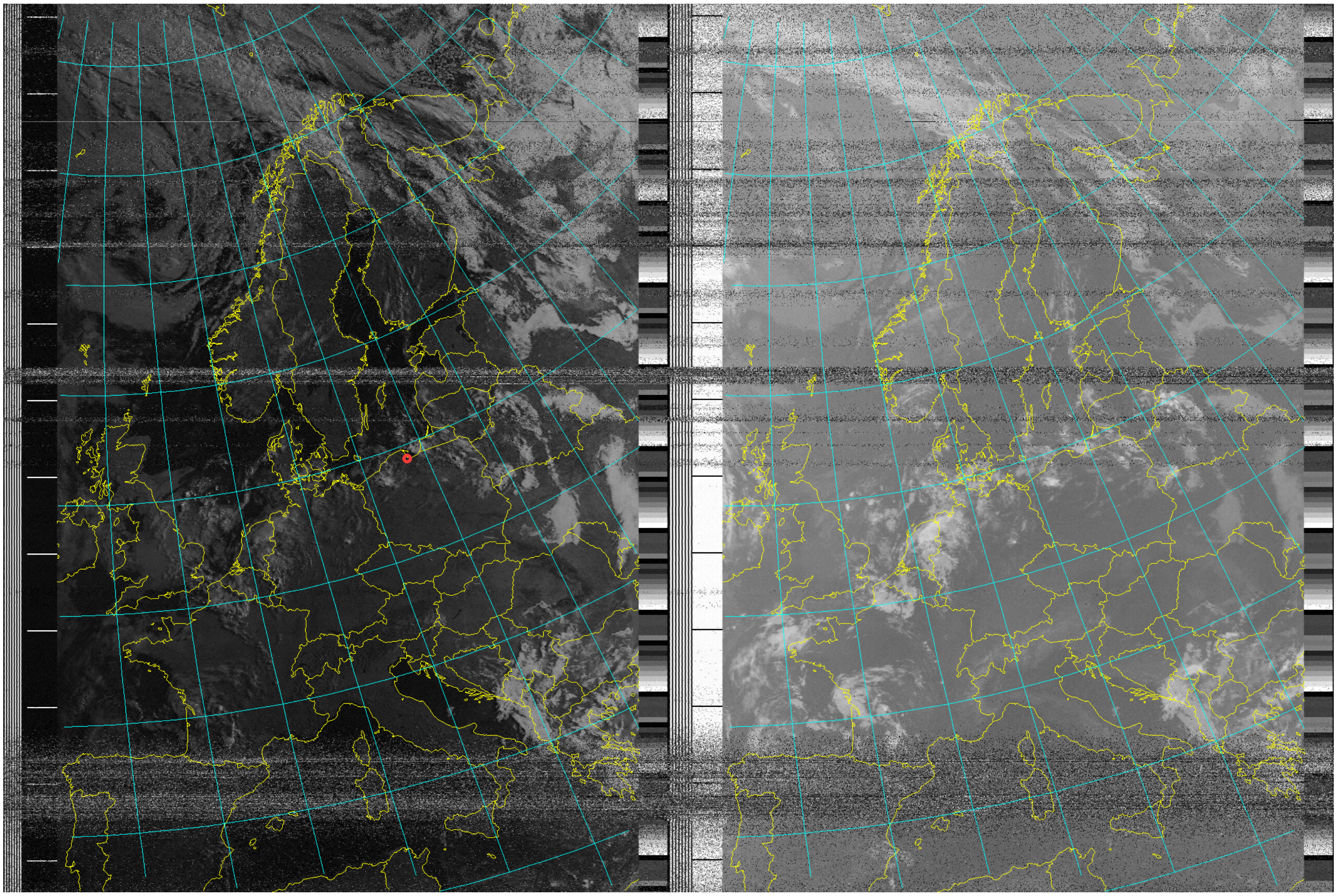

Figure 16: Georeferenced image from NOAA-19 satellite, data acquired in Satnogs project. The country countors (yellow) make the image much easier to recognize, especially in cases of high clould cover or faint contrast (right image). The geodetic grid (cyan) clearly shows the image distortion. Ground station location is shown in red.

Note: Pomysł na artykuł

Zaimplementowałem 3 algorytmy konwersji współrzędnych TEME na współrzędne geodezyjne oraz dwa sposoby konwersji tychże na współrzędne x,y obrazu. Istnieje kilka metod obliczania GMST. Wszystkie te alternatywy możnaby porównać w jaki sposób wpływają na uzyskaną precyzję końcowa. Precyzję można w łatwy sposób mierzyć. Można użyć różnych geoid (np. porównać WGS-72 z WGS-84) i ocenić, czy przy takiej skali obrazu w ogóle ma to jakikolwiek wpływ. Wystarczy na zdjęciu wybrać kilka punktów charakterystycznych (np. Hel, Gibraltar, czubek "buta" Włoch itd.) o znanych koordynatach, przeliczyć je na zdjęcie i zmierzyć błąd.

4.2. Problem 2: Reviving or deorbiting old satellites

Many older satellites still have fully functional electronic and optical systems, but they are no longer able to conduct their primary missions due to running out of fuel. A recent idea that seems to be particularly popular in NewSpace community is to come up with a revival sat – a satellite that would randezvous with existing 'dead' satellite, dock with it and then serve as a permanent new engine.

Designing such a mission would require several orbital manouvers: matching orbital plane, synchronizing apogee and perigee, and then performing approach and randezvous manouvers.

An alternative version of such a mission could attach to a dead satellite and conduct its deorbit. Cleaning up orbital debris is an activity that is strongly endorsed by ESA. It's also something that can effectively mitigate the threat of Kessler syndrome.

4.2.1. Orbital Manouvers

The orbital velocity is defined using the following formula:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) \tag{4.11}$$

where v is velocity, G is universal gravitational constant, M is mass of the object being orbited (e.g. Earth), r is the current distance between centers of the orbiting bodies and a is an average of semi-major and semi-minor axes. Since G and M are constant and occur frequently in many equations, they're conveniently replaced by a single constant:

$$GM = \mu \tag{4.12}$$

Furthermore, for circular orbits, $r = a$, so the equation simplifies to:

$$v = \sqrt{\mu \cdot \frac{1}{a}} \tag{4.13}$$

For example, we can calculate how much Δv is required to move an object from a circular LEO orbit with elevation of 300km to Geostationary orbit. The following assumes that the inclination of the starting orbit is zero (i.e. the orbit is on the equatorial plane). First, we need to calculate the orbital velocity for circular LEO orbit:

$$v_{LEO} = \sqrt{\mu \cdot \left(\frac{1}{Re + r}\right)} \approxeq 7723.29[m/s] \tag{4.14}$$

Re is Earth radius. Since we want the destination to be circular, we can calculate the velocity of the destination orbit:

$$v_{GEO} = \sqrt{\mu \cdot \left(\frac{1}{Re + 35786}\right)} \approxeq 3073.68[m/s] \tag{4.15}$$

In most cases, the orbital change is done using Hohmann transfers. For cases when alternatives are better, see Section 1.12 about bi-elliptic transfers. Hohmann transfer from circular to circular orbit is particularly easy, as Hohmann transfers are to be initiated during periapis or apoapsis. However, is the departing orbit is circular, any orbital position can be considered both apoapsis and periapsis. The first burn will turn the orbit into highly eliptical with periapsis of 300km and apoapsis of 35786km, often called GTO (Geostationary Transfer Orbit). Since this orbit is eliptical, it's velocity changes, depending on the spacecraft position. For periapsis and apoapsis it is respectively:

$$v_{p(LEO \Rightarrow GSO)} = \sqrt{\mu\left(\frac{2}{r_p} - \frac{2}{(r_p + r_a)}\right)} \approxeq 10148[m/s] \tag{4.16}$$

$$v_{a(LEO \Rightarrow GSO)} = \sqrt{\mu\left(\frac{2}{r_a} - \frac{2}{(r_p + r_a)}\right)} \approxeq 1607[m/s] \tag{4.17}$$

To move a spacecraft from LEO to GSO, the Δv required is $v_{p(LEO \Rightarrow GSO)} - v_{LEO}$, which is roughly $2424 m/s$. Once the spacecraft reaches its apogee, it should perform circularization manouver. It will have a velocity of 1607 m/s and needs to increase it to 3073 m/s, thus expending another 1466 m/s. By summing all manouvers together, we get the total Δv expenditure of the LEO (300km) to GEO transfer to be $3891 m/s$.

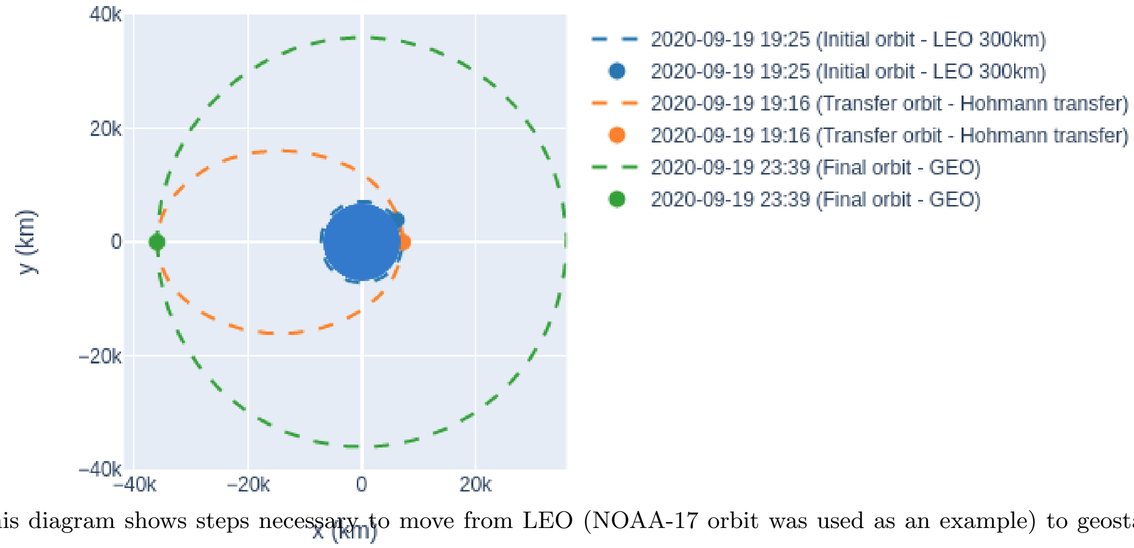

Figure 17: This diagram shows steps necessary to move from LEO (NOAA-17 orbit was used as an example) to geostationary orbit. Note that although both the original and destination orbits are circular, the intermediate Hohmann transfer orbit is eliptical. Generated using Poliastro software

The discussion above makes a strong assumption that both departure and destination orbits are coplanar. This greatly simplifies the calculations as velocity vectors are always in the same direction, thus can be added and subtracted as scalars.

4.2.2. Inclination Change

The general equation for Δv is a direct application of law of cosines, better knows as cosine rule.

$$\Delta V_2^2 = V_1^2 + V_2^2 - 2V_1V_2 \cdot cos(i) \quad (4.18)$$

The inclination change is a costly manouver. However, to better understand its cost, let's assume we want to do pure inclination change from one circular orbit to another circular orbit with the same radius. In this case $V_1 = V_2 = V$ and the equation can be simplified to:

$$\Delta V = V\sqrt{2 \cdot (1 - cos(i))} \quad (4.19)$$

For example, let's assume a scenario or launching a satellite from Łeba Proving Grounds aiming for equatorial LEO orbit of 300km. Łeba's latitude is 54°45'N. An orbital velocity of circular 300km LEO orbit is 7723.29 m/s. As such, the inclination change would require 7102 m/s. This is absurdly high value. As shown in earlier section, it is far easier to reach GEO orbit. This is the reason why launch locations located at high latitudes almost never launch to equatorial orbits and instead opt to specialize in polar orbits.

Let's hypotetically assume that Poland would build an launch complex at its sourthern most point, which is near Opołonek peak in Bieszczady mountains. Its latitude is 49°00'N. The Δv required from that location is 6405 m/s. It is still very unfavorable, but almost 700 m/s less than from Łeba. In any case, Poland is poorly situated for equatorial launches. As such, we as a nation have two possible development paths. First, we can focus on missions that use near-polar orbits as polar launches are easier to achieve from site with high latitudes. Second, for missions that require near-equatorial orbits (GEO and beyond Earth orbit), we cannot depend on domestic capabilities and must rely on third party launch sites.

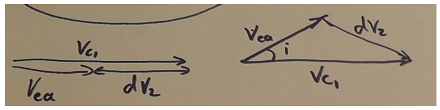

Figure 18: The difference between changing velocity in the same plane (no inclination change, left) and with inclination change (right). In general, it is always beneficial to conduct joint maneuvers together.

TODO: Narysować ten diagram porządnie.

4.2.3. Energy perspective

An alternative way to describe an orbit is to define the object's kinetic energy. As [4] shows, for a given orbit, the energy is dependent only on the major semi-axis:

$$E = -\frac{\mu}{2a} \tag{4.20}$$

It is worth noting the zero point in this notation. The mechanical energy of a satellite moving in a closed (circular or eliptical) orbit is negative, for parabolic orbits is zero and for hyperbolic orbits is positive.

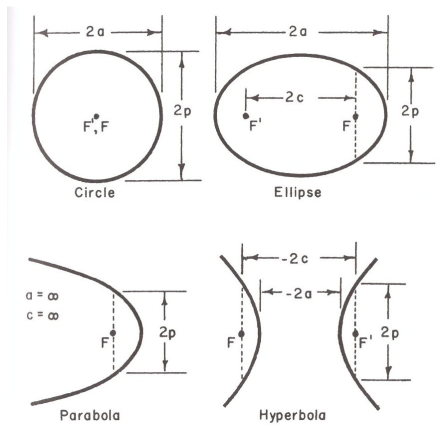

Figure 19: The dimensions that are common for all conic sections.

4.2.4. Escape velocity

[4]

Even though a gravitational field of a given body, such as Earth or Sun, extends to infinity, the strenght of it decreases rapidly and thus only finite amount of energy is required to escape it. The velocity needed to escape from a circular orbit of radius r is defined by the following equation:

$$V_{esc} = \sqrt{\frac{2\mu}{r}} \tag{4.21}$$

Based on this, we can formulate an interesting observation that for a any given circular orbit, the Δv necessary to escape is $v_{esc} = \sqrt{2} \cdot V_{cir}$, or rougly 1.414 more than the current orbital speed. This is intuitive. The higher an orbit is, the slower its actual velocity is. Therefore, the farther away from the central body it is, the easier it becomes to escape.

4.2.5. Rendezvous

The space randezvous is a set of orbital manouvers where two spacecraft physically touch or get in very close proximity with very small relative velocity. A randezvous requires matching orbital velocities and orbital vectors. There are many practical applications for this manouver: docking or berthing a spaceship to a station, attaching to a new satellite to an old one, assembling larger structures in space and many more. Also, for some bodies with small mass, such as Mars' moons of Phobos and Deimos, the manouver of landing on their surface looks more like rendezvous, rather than typical entry, descent and landing manouvers.

The rendezvous is counte-intuitive and requires good understanding of astrodynamics. For example, imagine situation depicted in Fig. 20. There is a spacestation and a ship attempting to dock to it. Let's assume that both ship and the station are on circular orbit of the same radius r. What manouvers should the ship conduct to dock? The intuitive answer – to fire its thrusters directly towards the station – is incorrect. By firing its thrusters in the prograde direction, the ship will extend the opposite point of the orbit, thus effectively moving to an eliptic orbit with periapis equal to r, but its apoapsis being $r + \Delta h$. As the ship and station travel along their orbits, the ship would slowly drift away to a higher altitude, and after half an orbit it would reach a maximum distance of Δh from its previous orbital path. After completing the full orbit, it would return to its original altitude and distance to the station.

This is the mistake that US astronaut Jim McDivitt made when attempting to perform first rendezvous manouver in history. He tried to maneuver his Gemini 4 capsule to match the Titan II's upper stage. Despite multiple attempts, the manouver was unsuccessful. [29].

Figure 20: One object, such as a capsule, approaching another object, such as a space station, needs to perform rendezvous manouver to dock or berth.

The proper way to approach the target is to move the approaching ship to a lower orbit. Lower orbit has a higher orbital velocity, so the chasing ship will "catch up" with the target. Once it is close, it should then raise its own orbit to that of the target's. This is counter-intuitive as the first manouver (move to lower orbit) require firing engines in the retrograde direction, i.e. away from the target.

The various phases of rendezvous are well described in [24].

4.3. Problem 3: Are we all gonna die on April 29th 2020? – Debunking fake news

Many newspapers are publishing articles about the asteroid 1998 OR-2 passing very close by to Earth, with varying levels of inaccuracy and fake sensationalism. I'd like to present a step by step explanation how to calculate ephemerides for the upcoming randezvous, including current known uncetainties and compare it to predicted closest distance.

Comment: spoiler alert: Nie, nie zginiemy.

4.4. Problem 4: Interplanetary Transfer Windows

The difficulty of reaching a place in a Solar system is not expressed in distance, but in the relative change of velocity. Due to the bodies being in constant movement, the difficulty changes over time. There are certain configurations where reaching one planet from another are more easier. Such periods of favorable configurations are called transfer windows. For example, the transfer windows for Earth-Mars are open roughly once every two years. One of the goals here will be to calculate charts for choosing optimal departure times. Such calculations can be repeated for many destinations in the Solar System, and a delta-v map can be computed.

Comment: Ten problem mam w miarę dobrze opanowany. Umiem liczyć okna transferowe, ale na razie brakuje mi opisów, co dokładnie sie dzieje. Wrzucam na razie kilka screenshotów dla okien startowych na Marsa i Wenus w niedalekiej przyszłości: 21, 22, 23, 24

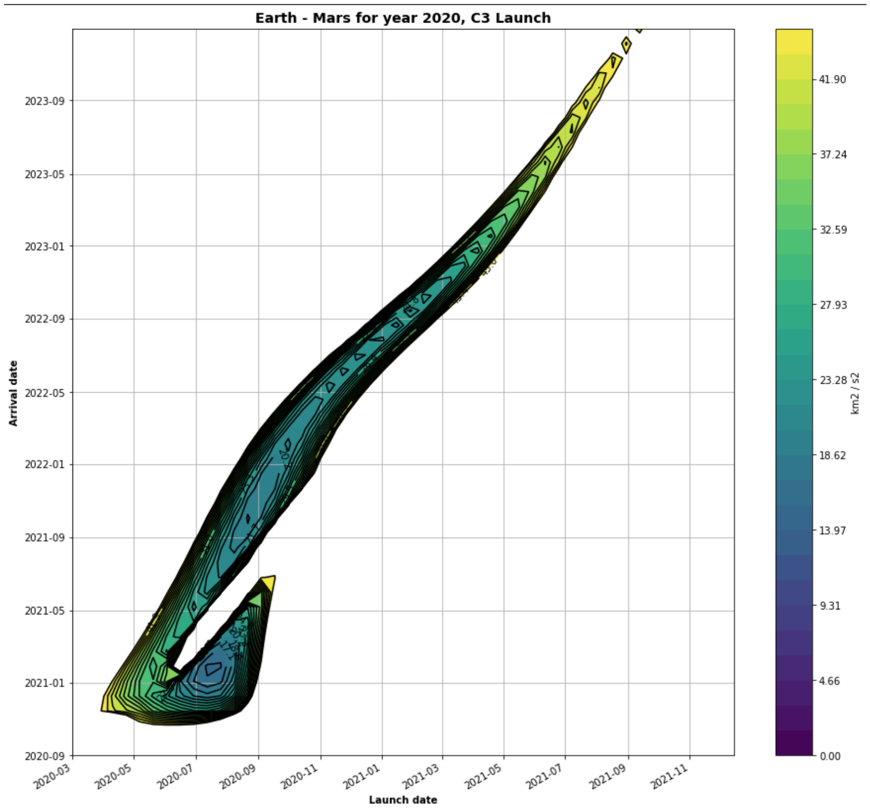

Figure 21: This diagram shows the full Mars transfer window, for departure in 2020-2021 and arrival in 2021-2022. A close-up of the most optimal trajectories is shown in the next Figure.

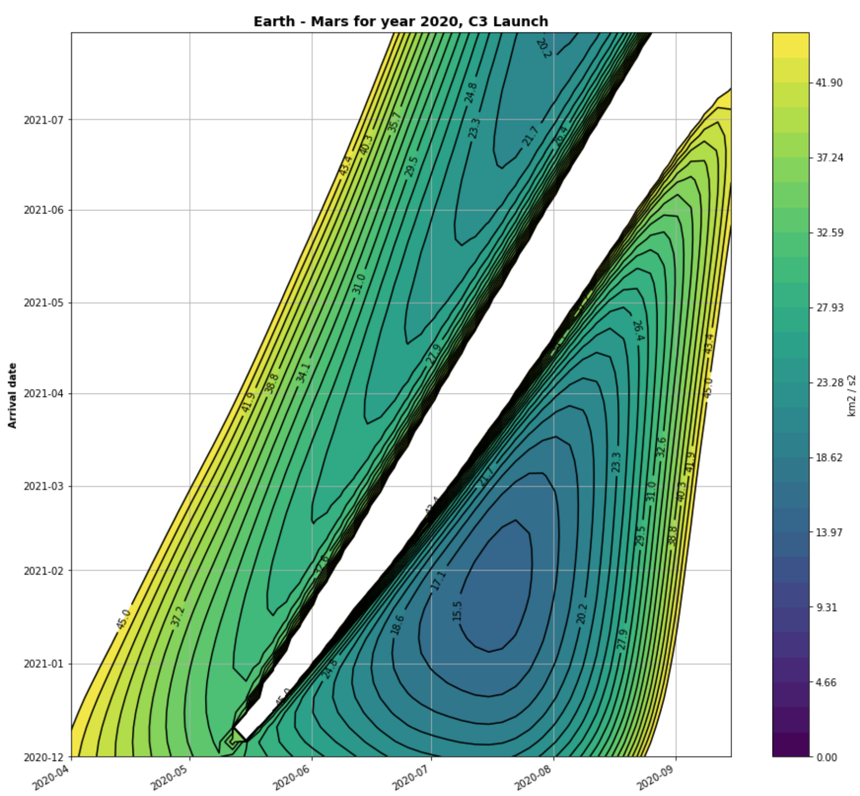

Figure 22: A close-up of the optimal trajectories region for Earth-Mars transfer window.

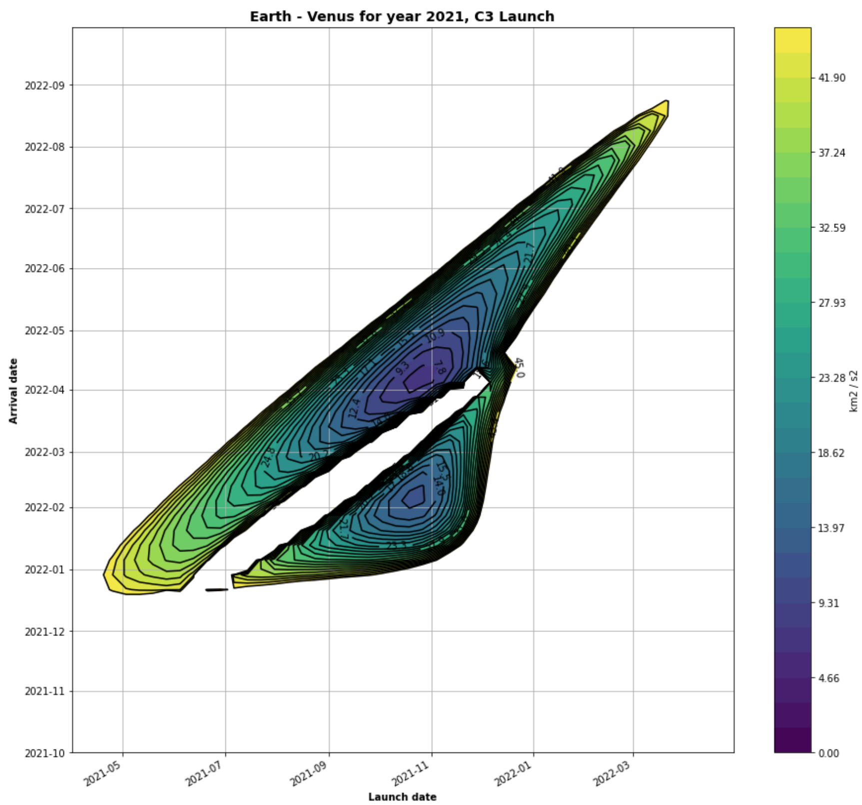

Figure 23: This diagram shows the full Venus transfer window, for departure in 2021-2022 and arrival in 2022. A close-up of the most optimal trajectories is shown in the next Figure.

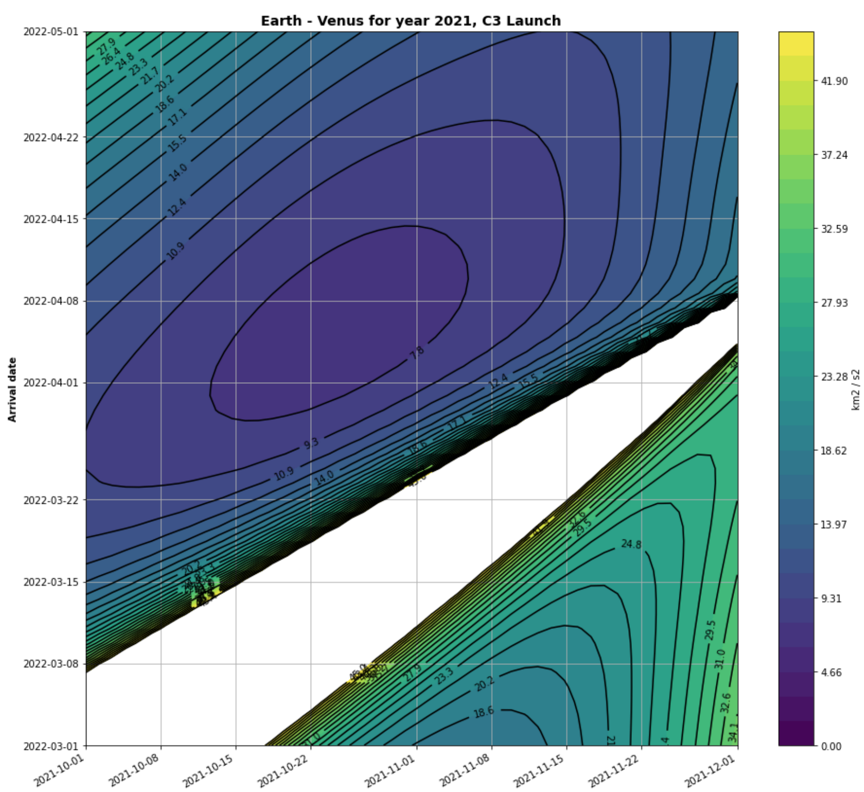

Figure 24: A close-up of the optimal trajectories region for Earth-Venus transfer window.

4.5. Problem 5: Asteroid Intercept Mission Proposal

As of today, there are close to 800 000 asteroids known in the Solar System. Many of them belong to a NEA (Near Earth Asteroid) class. The goal of this problem is to review existing known NEA asteroids, pick one or several as target candidates and then calculate necessary manouvers needed for a probe to leave LEO and intercept the target as it flys by close to the Earth.

This problem brings in additional complexity of reaching Earth escape velocity, changing frame of reference to heliocentric, changing inclination and other orbital parameters to match those of the target.

4.6. Problem 6: Navigating with low force engine cubesat

The economic reality implies that Poland is currently incapable of launching any satellites larger than cubesats. This form factor is too small to have any conventional chemical propulsion. However, there are several possible alternative propulsion mechanisms that can be taken into consideration. One of them is a solar sail that uses solar radiation pressure for small, but constant acceleration.

A solar sail could possibly be used to perform some manouvers, such are raising perygee and apogee of the orbit. One complication is that the force vertor always points directly outwards from the Sun. This would imply the sail would have to change orientation in various sections of its orbit around Earth. That, however, should be doable with magnetotorquer, a clever mechanism that generates magnetic dipole that interfaces with Earth's magnetic field, thus providing torque and eventually rotating the spacecraft.

The goal of this problem is to propose a CubeSat mission that would use a solar sail for actual navigation. In a sense, such a mission would be a follow-up to PW-Sat 2 mission that proved that small solar sails can be deployed and used in space.

4.7. Problem 7: Website for observing Polish Satellites

Poland has launched six satellites: LEM, Heweliusz, Swiatowid, PW-Sat, PW-Sat 2 and KRAKSat. There are also some satellites currently in orbit that have instruments on-board that were built in Poland. A website that predicts fly-overs of Polish satellites could be developed for educational purposes. The site would have to calculate upcoming fly-overs, especially with the preferred condition for satellite visibility. The observing site conditions should be in the dark (after sunset or before sunrise) and the satellite flying outside of the Earth shadow.

The goal would be to increase public interest in Polish space activities.

5. Solution evaluation

This chapter describes the process used to validate the developed software. Currently three verification methods are taken into consideration. The first one is to compare the calculations with the results generated by existing, well known software packages, such as STK, GMAT or SGP4 models. Second validation is based on the concept of unit and system tests, a software development regime developing tests and software as two interdependent elements. The third validation to be considered is to conduct actual observations and compare measurements with the models. The author has access to astronomical equipment and an attempt could be made to measure some parameters empirically.

6. Conclusions

This final chapter offers summary of discussed topics, provides conclusions regarding .... Summary of all achievements and directions of future work conclude this dissertation.

During the course of numerous experiments and research conducted by the author, number of conclusions emerged. Main results obtained in this dissertation clearly prove that:

1. ...

In the light of the presented achievements, it can be stated that the thesis: **abc** has been proven.

6.1. Practical aspects

...

6.2. Future work

As this research proved validity of several proposed solutions, the next logical step is to ...

Bibliography

[1] D. Vallado, P. Crawford, R.Hujsak, T.S.Kelso, "Revisiting Spacetrack Report #3", AIAA Astrodynamics Specialist Conference, 2006

[2] R. Makówka, "NASA ostrzega: W stronę Ziemi pędzi asteroida. Koniec świata może nastąpić 29 kwietnia 2020?", https://www.antyradio.pl/News/NASA-ostrzega-W-strone-Ziemi-pedzi-asteroida-Koniec-swiata-moze-nastapic-29-kwietnia-2020-39951, AntyRadio, 2020-03-23

[3] E. Aldrin, "Line-of-sight guidance techniques for manned orbital randezvous", MIT press, 1963

[4] R. Bate, D. Mueller, J. White, "Fundamentals of astrodynamics", Dover Publications, 1971

[5] Konstantin Ciołkowski, "Exploring world spaces with rocket devices", 1903, "Selected works of Konstantin Tsiolkovsky", Univ. of Hawaii, 2004

[6] H. Curtis, "Orbital Mechanics for Engineering Students", Elsevier, 2010

[7] R.W. Farquhar, "The control and use of libration-point satellites", PhD thesis, Stanford university, 1968

[8] D. Izzo, "Revisiting Lambert's problem", https://www.esa.int/gsp/ACT/doc/MAD/pub/ACT-RPR-MAD-2014-RevisitingLambertProblem.pdf, retrieved 2020-04-26

[9] T.S. Kelso, "Orbital Coordinate Systems, Part III", Satellite Times, Jan. 1996, https://celestrak.com/columns/v02n03/

[10] J. Kepler, "The Harmonies of the World", 1619, Encyclopaedia Britannica, published 1952

[11] M. Kopernik, "De revolutionibus orbium coelestium" (pol. "O obrotach sfer niebieskich"), 1543, published 1953

[12] M. Bloch, Mapshape service, https://github.com/mbloch/mapshaper, retrieved Aug. 2020

[13] J. J. Retyk, "Narratio Prima", Fundacja Nicolaus Copernicus, 2015

[14] E. Mahoney, "NASA Selects Blue Origin, Dynetics, SpaceX for Artemis Human Landers", https://www.nasa.gov/feature/nasa-selects-blue-origin-dynetics-spacex-for-artemis-human-landers, NASA, April 2020

[15] N. Kelso, T.Patterson, Natural Earth, https://www.naturalearthdata.com/downloads/10m-cultural-vectors/10m-admin-0-countries/, retrieved on July 2020

[16] I. Newton, "Philosophiæ Naturalis Principia Mathematica", 1687, polish edition, Copernicus Center Press, 2017

[17] M. Bernardi, "The NOAA-APT project: How it work?", https://noaa-apt.mbernardi.com.ar/, retrieved Aug. 2020

[18] C. Bailer-Jones, D. Farnocchia, Q. Ye, K. Meech, M. Micheli, "A search for the origin of the interstellar comet 2I/Borisov", Astronomy & Astrophysics, ESO 2020

[19] Claudius Ptolemy, "Almagest", 140CE, Greece, published by University of Chicago 1952

[20] T.Mrugalski, S.Figiel, "Satnogs-GDN project", https://satnogs.klub.com.pl, retrieved Aug 2020

[21] NASA, "Definition of Two-line Element Set Coordinaate System", https://spaceflight.nasa.gov/realdata/sightings/SSapplications/Post/JavaSSOP/SSOP_Help/tle_def.html, retrieved 2020-04-26

[22] D. Vallado, P. Cefola, “Two-Line Element Sets – Practice and Use”, International Astronautical Congress, IAC-12-A6.6.11, 2012

[23] W. von Braun, “Project Mars, a technical tale”, 1949, published by Ishi Press, 2017

[24] J. Wertz, R. Bell, “Autonomous Rendezvous and Docking technologies – Status and Prospects”, Space Systems Technology and Operations Conference, Orlando, 2003

[25] Wikipedia, “Orbital Mechanics”, https://en.wikipedia.org/wiki/Orbital_mechanics, retrieved 2020-08-02

[26] Wikipedia, “Prędkość ucieczki”, (pol. Escape velocity), https://pl.wikipedia.org/wiki/Pr%C4%99dko%C5%9B%C4%87_ucieczki, retrieved 2020-09-25

[27] Wikipedia, “The United States Department of Defense World Geodetic System 1972”, Wikipedia, https://en.wikipedia.org/wiki/World_Geodetic_System#The_United_States_Department_of_Defense_World_Geodetic_System_1972, retrieved on 2020-08-30

[28] Wikipedia, “Lissajous orbit”, https://en.wikipedia.org/wiki/Lissajous_orbit, retrieved 2020-09-27

[29] Wikipedia, “Space rendezvous, First attempt failed”, Wikipedia, https://en.wikipedia.org/wiki/Space_rendezvous#First_attempt_failed, retrieved on 2020-09-18

[30] Wikipedia, “Sun-synchronous orbit”, https://en.wikipedia.org/wiki/Sun-synchronous_orbit, retrieved 2020-09-27

[31] E. Zimovan, K. Howell, D. Davis, “Near Rectilinear Halo Orbits and their application in cis-lunar space”, 3rd IAA Conference on Dynamics and Control of Space Systems, 2017

Vocabulary

Spis terminów